Royal Commission for Jubail and Yanbu

Jubail University College

<u>Department of Mechanical Engineering</u>

PROBABILITY & STATISTICS

DR. NEHRU

MATH 312

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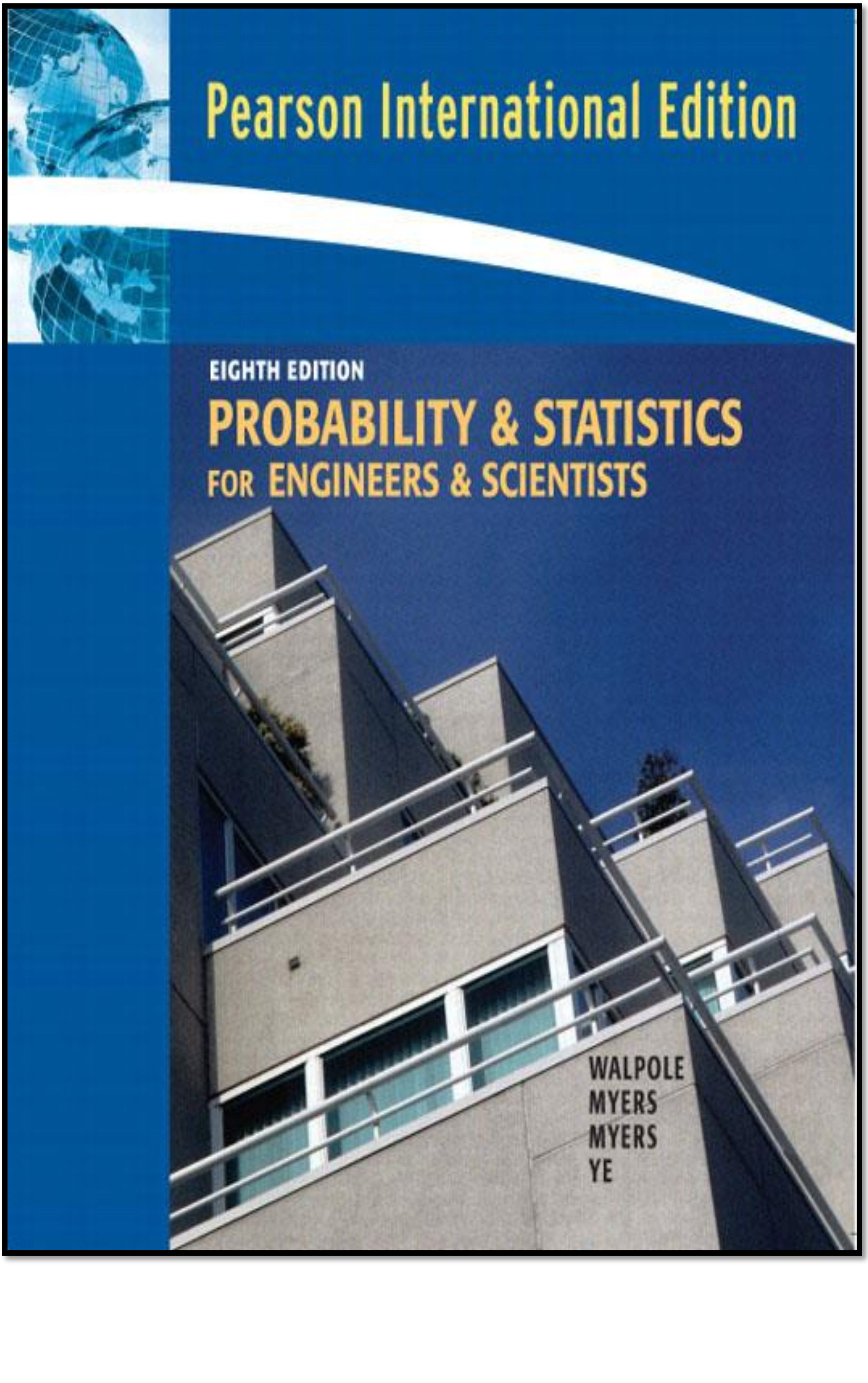
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Chapter 1

Ungrouped Data

- Mean — $\sum \frac{xi}{n}$
- Median
  - ⇨ In odd numbers we take the mid number
  - ⇨ In even numbers we take the 2 mid numbers and calculate the average
- Mode
  - ⇨ Take the most repeated number
  - ⇨ No Mode ; if no number repeated
- Range — Highest num − Lowest num
- Mid Range — $\frac{\text{Highest num} + \text{Lowest num}}{2}$
- Variance — $S^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{\sum x^2 - \sum \frac{x^2}{n}}{n-1}$
- Standerd Deviation — $S = \sqrt{S^2}$

Grouped Date

- Mean — $\sum \frac{Rf}{fi}$
- Median — *Location of Median* $= \left[\frac{\sum fi + 1}{2}\right]$ — $= L + \left[\frac{\sum \frac{fi}{2} - Cf}{f}\right](i)$
- Mode — The Mid point of the highest frequancy value
- Range — Highest num − Lowest num
- Mid Range — $\frac{\text{Highest num} + \text{Lowest num}}{2}$
- Variance — $S^2 = \frac{\sum fxi^2 - \sum \frac{(fxi)^2}{fi}}{\sum fi - 1}$
- Standerd Deviation — $S = \sqrt{S^2}$

▪**Example** : Group A : 1,2,2,2,3,3,4 Group B : 1,2,3,4

Find Mean , Median , Mode , Range , Mid Range for two groups ?

Group A	Group B
$\bar{x} = \frac{1+2+2+2+3+3+4}{7} = 2.42$	$\bar{x} = \frac{1+2+3+4}{4} = 2.5$
Median = 2	Median $= \frac{2+3}{2} = 2.5$

Mode = 2	No Mode
Range = 4 – 1 = 3	Range = 4 – 1 = 3
Mid Range $= \frac{4+1}{2} = 2.5$	Mid Range $= \frac{4+1}{2} = 2.5$

▪**Example** : Given 1,2,2,2,3,3,4 Find Variance , Standard deviations ?

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\bar{x} = 2.42$
1	-1.4	1.96	$\sum (x - \bar{x})^2 = 5.72$ $S^2 = \frac{5.72}{6} = 0.953$
2	-0.4	0.16	
2	-0.4	0.16	
2	-0.4	0.16	
3	-0.6	0.36	$S = \sqrt{0.953} = 0.976$
3	-0.6	0.36	
4	1.6	2.56	

▪**Example①**: Given 1,2,3,4,5,6,7,8,8,9 Find 10% of Trimmed Mean ?

We take 10% from these numbers = 1 from beginning and 1 from end

After trimmed = 2,3,4,5,6,7,8,8 , $\bar{x} = \frac{2+3+4+5+6+7+8+8}{[10-2(1)]=8} = 5.375$

▪**Example②**: Given 1,2,3,4,5,6,7,8,8,9 Find 15% of Trimmed Mean ?

We take 15% from these numbers = $\frac{15}{100} \times 10 = 1.5$ from beginning and 1.5 from end

After trimmed = 1,3,4,5,6,7,8,4 , $\bar{x} = \frac{1+3+4+5+6+7+8+4}{[10-2(1.5)]=7} = 5.42$

▪**Example③**: Given 1,2,3,4,5,6,7,8,8,9 Find 12% of Trimmed Mean ?

We take 12% from these numbers = $\frac{12}{100} \times 10 = 1.2$ from beginning and 1.2 from end

After trimmed = 0.4,3,4,5,6,7,8,1.6 , $\bar{x} = \frac{0.4+3+4+5+6+7+8+1.6}{[10-2(1.2)]=7.6} = 4.60$

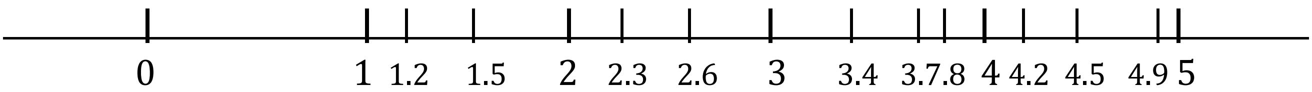
▪**Example④**: Given 1,2,3,4,5,6,7,8,9 Find 10% of Trimmed Mean ?

We take 12% from these numbers = $\frac{10}{100} \times 9 = 0.9$ from beginning and 0.9 from end

After trimmed = 0.9,3,4,5,6,7,8,8.1 , $\bar{x} = \frac{0.9+3+4+5+6+7+8+8.1}{[9-2(0.9)]=7.2} = 4.60$

▪**Example⑤**: Given 1.2 , 2.3 , 3.4 , 1.5 , 2.6 , 3.8 , 4.9 , 3.8 , 3.7 , 4.2 , 4.5

A) plot the data ?



B) represented stem and leaf ?

Stem	Leaf	Frequency
1	2 , 5	2
2	3, 6	2
3	4 , 8 , 8 ,7	4
4	9 , 2 , 5	3
Key 1/2 = 1.2		Total = 11

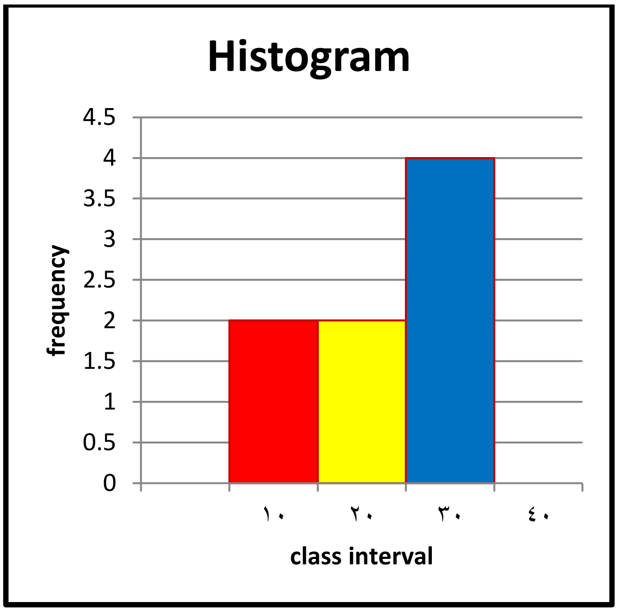
▪**Example⑥:** group A :101 , 102 , 103 , 104 and group B : 124 , 114 , 113 , 119 represented stem and leaf ?

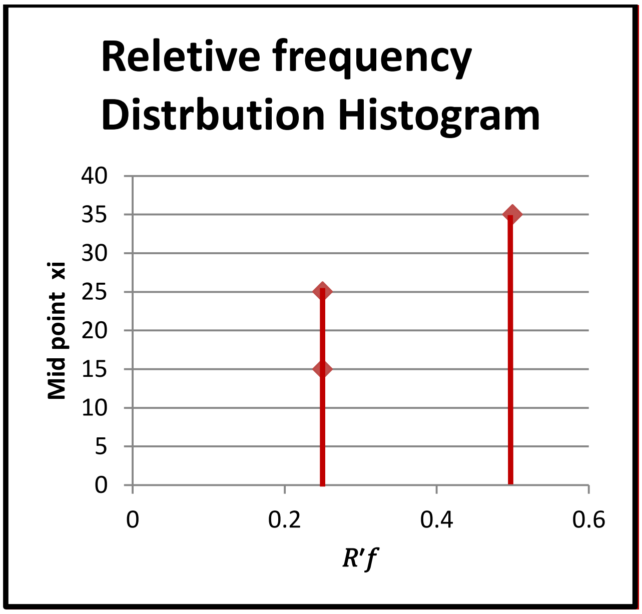
Frequency	Leaf	Stem	Leaf	Frequency
4	24 , 14 , 13 , 19	1	01 , 02 , 03, 04	4

▪**Example⑦:** Given 10 , 12 , 25 , 30 , 32 , 35 , 39

Draw the histogram and the relative frequency distribution histogram ?

Class	Frequency	Mid point	Relative Frequency Distribution
10 – 20	2	15	$2 \div 8 = 0.25$
20 – 30	2	25	$2 \div 8 = 0.25$
30 – 40	4	35	$4 \div 8 = 0.5$
	Total = 8		Total = 1





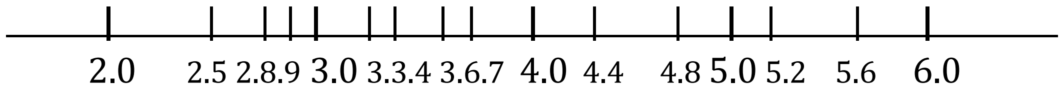
Exercises

1.1 Page 13 : **A)** 15

B) $\bar{x} = \frac{3.4+2.8+4.4+2.5+3.3+4.0+4.8+5.6+5.2+2.9+3.7+3.0+3.6+2.8+4.8}{15} = 3.78$

C) Median = 5.6

D)



E) first we arrange : 2.5,2.8,2.8,2.9,3.0,3.3,3.4,3.6,3.7,4.0,4.4,4.8,4.8,5.2,5.6

$\frac{20}{100} \times 15 = 3$ After trimmed = 2.9 , 3.0 , 3.3 , 3.4 , 3.6 , 3.7 , 4.0 , 4.4 , 4.8

$= \frac{2.9+3.0+3.3+3.4+3.6+3.7+4.0+4.4+4.8}{9} = 3.67$

1.2 Page 13 :

Class	f	xi	fxi	fxi^2	Cf
$18.0 - 19.0$	3	18.5	$18.5 \times 3 = 55.5$	$18.5^2 \times 3 = 1026.75$	3
$19.0 - 20.0$	4	19.5	$19.5 \times 4 = 78$	$19.5^2 \times 4 = 1521$	$3 + 4 = 7$
$20.0 - 21.0$	4	20.5	$20.5 \times 4 = 82$	$20.5^2 \times 4 = 1681$	$7 + 4 = 11$
$21.0 - 22.0$	4	21.5	$21.5 \times 4 = 86$	$21.5^2 \times 4 = 1849$	$11 + 4 = 15$
$22.0 - 23.0$	3	22.5	$22.5 \times 3 = 67.5$	$22.5^2 \times 3 = 1518.75$	$15 + 3 = 18$
$23.0 - 24.0$	2	23.5	$23.5 \times 2 = 47$	$23.5^2 \times 2 = 1104.5$	$18 + 2 = 20$
	20		Total = 416	Total = 8701	

⇨ xi : *Mid point* ; f: *frequency* ; fxi : *Reletive frequency* ;
Cf: *Clumulative frequency*;

a) $\bar{x} = \frac{416}{20} = 20.8$; *Location of Median* $= \left[\frac{20+1}{2}\right] = 10.5$

$$Median = L + \left[\frac{\frac{\sum fi}{2} - Cf}{f}\right](i) = 20 + \left[\frac{\frac{20}{2} - 7}{4}\right](1) = 20.75$$

⇨ L: *lower limit in the Median class* ; Cf: *Cf before MC* ; f: *frequency of MC* ;
i : *class interval size*

1.7 Page 17 :

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\bar{x} = 3.78$
2.5	- 1.28	1.63	$\sum (x - \bar{x})^2 = 13.108$ $S^2 = \frac{13.108}{14} = 0.936$
2.8	-0.98	0.96	
2.8	-0.98	0.96	
2.9	-0.88	0.77	
3.0	-0.78	0.60	
3.3	-0.48	0.23	
3.4	-0.38	0.14	
3.6	-0.18	0.03	
3.7	-0.08	6.4×10^{-3}	
4.0	0.22	1.6×10^{-3}	$S = \sqrt{0.936} = 0.967$
4.4	0.62	0.38	
4.8	1.02	1.04	
4.8	1.02	1.04	
5.2	1.42	2.01	

5.6	1.82	3.31	

1.13 Page 28 :

A) $\bar{x} = \frac{123+116+122+110+175+126+125+111+118+117}{10} = 124.3$

Median $= \frac{175+126}{2} = 150.5$

1.14 Page 28 :

A) $\bar{x} = \frac{572+572+573+568+569+575+565+570}{8} = 570.5$

Median $= \frac{568+569}{2} = 568.5$

B) Range $= 575 - 565 = 10$

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\bar{x} = 570.5$
572	1.5	2.25	$\sum(x - \bar{x})^2 = 70$
572	1.5	2.25	
573	2.5	6.25	
568	-2.5	6.25	$S^2 = \frac{70}{7} = 10$
569	-1.5	2.25	
575	4.5	20.25	
565	-5.5	30.25	$S = \sqrt{10} = 3.16$
570	-0.5	0.25	

1.18 Page 28 :

Class	f	xi	fxi	$R'f$	fxi^2	Cf
$10 - 20$	3	15	$15 \times 3 = 45$	$3 \div 60 = 0.05$	$15^2 \times 3 = 675$	3
$20 - 30$	2	25	$25 \times 2 = 50$	$2 \div 60 = 0.03$	$25^2 \times 2 = 1250$	5
$30 - 40$	3	35	$35 \times 3 = 105$	$3 \div 60 = 0.05$	$35^2 \times 3 = 3675$	8
$40 - 50$	4	45	$45 \times 4 = 180$	$4 \div 60 = 0.06$	$45^2 \times 4 = 11.25$	12
$50 - 60$	5	55	$55 \times 5 = 275$	$5 \div 60 = 0.08$	$55^2 \times 5 = 15125$	17
$60 - 70$	11	65	$65 \times 11 = 715$	$11 \div 60 = 0.18$	$65^2 \times 11 = 46475$	28
$70 - 80$	14	75	$75 \times 14 = 1050$	$14 \div 60 = 0.23$	$75^2 \times 14 = 78750$	42
$80 - 90$	14	85	$85 \times 14 = 1190$	$14 \div 60 = 0.23$	$55^2 \times 5 = 101150$	56
$90 - 100$	4	95	$95 \times 4 = 380$	$4 \div 60 = 0.06$	$95^2 \times 4 = 36100$	60
	60		Total =3990	Total = 0.97	Total = 291300	

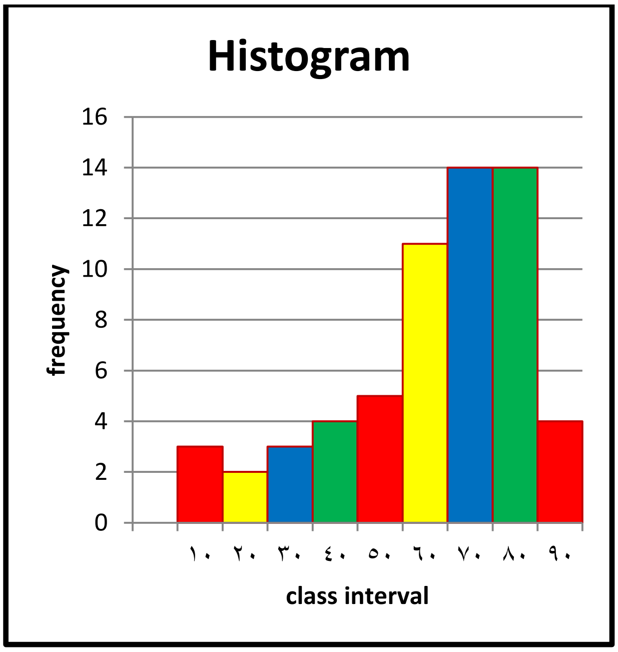
⇨ $R'f: Reletive\ frequency\ distribution$

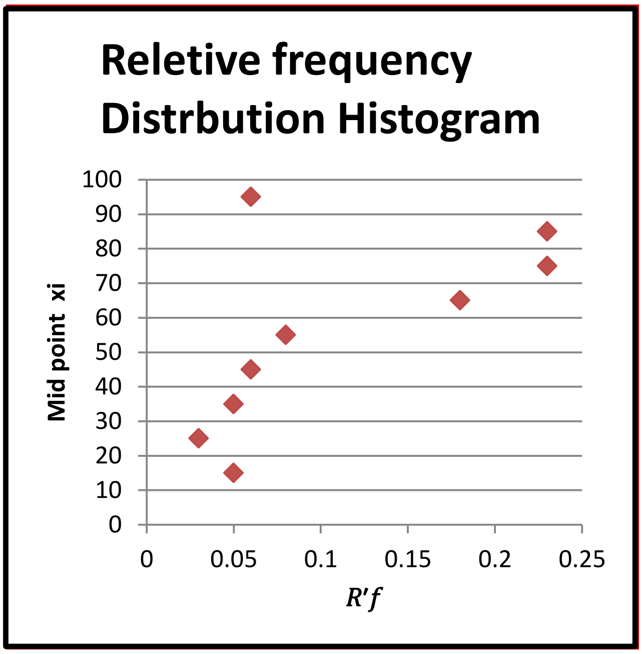
D) $\bar{x} = \frac{3990}{60} = 66.5$; $Location\ of\ Median = \left[\frac{60+1}{2}\right] = 30.5$

$$Median = L + \left[\frac{\sum\frac{fi}{2} - Cf}{f}\right](i) = 60 + \left[\frac{\frac{60}{2} - 17}{11}\right](10) = 71.81$$

$$S^2 = \frac{\sum fxi^2 - \sum\frac{(fxi)^2}{fi}}{\sum fi - 1} = \left[\frac{291300 - \left(\frac{15920100}{60}\right)}{59}\right] = 440.08$$

C)





1.21 Page 29 :

Class	f	xi	fxi	fxi^2	Cf
$0.5-1$	2	0.75	$0.75\times 2=1.5$	$0.75^2\times 2=1.125$	2
$1-1.5$	5	1.25	$1.25\times 5=6.25$	$1.25^2\times 5=7.81$	$2+5=7$
$1.5-2$	23	1.75	$1.75\times 23=40.25$	$1.75^2\times 23=70.43$	$7+23=30$
$2-2.5$	9	2.25	$2.25\times 9=20.25$	$2.25^2\times 9=45.56$	$30+9=39$
$2.5-3$	1	2.75	$2.75\times 1=2.75$	$2.75^2\times 1=7.56$	$39+1=40$
	40		Total =71	Total = 132.485	

A) $\bar{x}=\frac{71}{40}=1.775$; $Location\ of\ Median=\left[\frac{40+1}{2}\right]=20.5$

$$Median=L+\left[\frac{\sum\frac{fi}{2}-Cf}{f}\right](i)=1.5+\left[\frac{\frac{40}{2}-7}{23}\right](0.5)=1.78$$

B) $S^2=\frac{\sum fxi^2-\sum\frac{(fxi)^2}{fi}}{\sum fi-1}=\left[\frac{132.485-\left(\frac{5041}{40}\right)}{39}\right]=0.165$; $S=\sqrt{0.165}=0.40$

Chapter 2

▪**Sample Size (S)** : is the set of all possible outcomes of statistical experiment .
We can write $S = \{1 ,2 ,3 ,4 ,5 ,6 \}$ or $S = \{x \mid 1 \leq x \leq 6 \}$

▪**Complement (c or ')** : is the subset of all elements of S that are not in specific group .

If $S = \{H ,T \}$ so A' or $A^c = \{T\}$ H : Heads , T : Tail

The possibility for heads is $= \left\{\frac{1}{2}\right\}$ The possibility for tail is $= \left\{\frac{1}{2}\right\}$

$\sum P_i = 1$ from the possibilities for H and T $= \left\{\frac{1}{2} + \frac{1}{2}\right\}$, $0 < P_i < 1$

▪**Example①**: Given $S = \{1 ,2 ,3 ,4 ,5 ,6 \}$ if $A = \{1 , ,5 ,6 \}$ Find A^c and $A^c \cap A$ and $(A^c)^c$?

▪$A^c = \{ 2 ,3 ,4 \}$, $A^c \cap A = \emptyset$, $(A^c)^c = \{1 ,2 ,3 ,4 ,5 ,6 \}$

▪**Permutation** : The number of permutation of n distinct objects taken r at a time is $nP_r = \frac{n!}{(n-r)!}$

▪**Circle Permutation** : is $(n - 1)!$

▪**Combination** : is $nC_r = \frac{n!}{r!(n-r)!}$

▪**Example②**: **A)** How many of distinct 2 permutations made from the word CAT ? **B)** How many of distinct permutations made from the word CAT ?
C) How many of distinct 2 combinations made from the word CAT ?

A) $3\,P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 6$

B) $3\,P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3 \times 2 \times 1}{1} = 6$

C) $3\,C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2! \times 1!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$

▪**Example③**: How many distinct permutations made from the word COLLEGE ?

$if\ we\ have\ repeated\ letters\ we\ use\ this\ rule$ $\frac{n!}{n1!\ n2!\ n3!\ \ldots\ldots\ nk!}$

$$\frac{7!}{1!1!\ 1!\ 2!\ 2!} = \frac{7!}{1 \times 1 \times 1 \times 2 \times 2} = \frac{7!}{4} = 1260$$

▪The probability of an event A is the sum of the weights of all sample points in A
Therefore : $0 \leq P \leq 1$, $P(\emptyset) = 0$, $P(S) = 1$

$$\left[\frac{number\ of\ success}{Total\ number\ of\ events}\right]$$

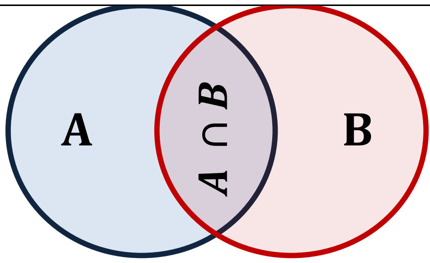
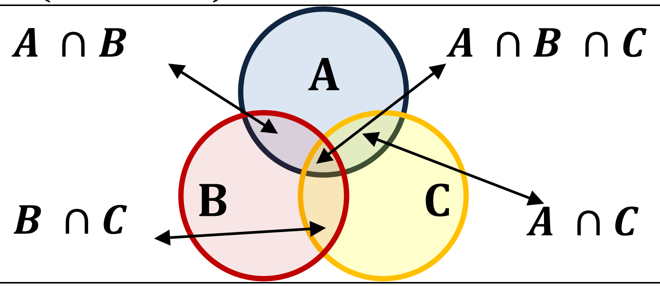
▪**Example④**: When a coin is tossed twice , Find the probability of getting at least one tail and one head ?

$S = \{ HH , HT , TT , TH \}$

Probability (getting one tail) = { HT , TT , TH } $= \frac{3}{4}$

Probability (getting one head) = { HH , HT , TH } $= \frac{3}{4}$

▪**Additive Rule** :

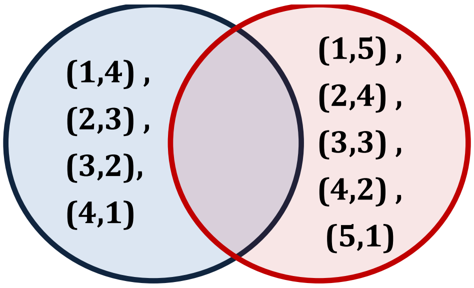
A , B are two events	A , B , C are three events
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
	
if A & B *dis joint* $A \cap B = \emptyset$ $P(A \cup B) = P(A) + P(B)$	*if* A & B & C *dis joint* $A \cap B \cap C = \emptyset$ $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

▪**Example⑤**: Find the probability of getting sum 5 or sum 6 when tossing a pier of dice ?

Probability (sum 5) = { (1,4), (2,3), (3,2), (4,1) } $= \frac{4}{36} = \frac{1}{4} = 0.25$

Probability (sum 6) = { (1,5), (2,4), (3,3), (4,2) , (5,1)} $= \frac{5}{36} = 0.13$

P (sum 5) ∪ P(sum6) = { 0.25 + 0.13 } = 0.38



▪**Conditional Probability** : $P(B|A) = \frac{P(A \cap B)}{P(A)}$

▪**Example⑥**: According this table .

A) Find the probability of male when he is already employed ?

B) Find the probability of Female when she is already un employed ?

C) Find the probability of already employed from male ?

	Employed	Un Employed	Total
Male	12	14	26
Female	10	15	25
Total	22	29	51

$P(Male) = \frac{26}{51}$, $P(female) = \frac{25}{51}$, $P(Employed) = \frac{22}{51}$, $P(Un) = \frac{29}{51}$

A) $P\,(M\,|E\,) = \frac{P\,(M\,\cap E\,)}{P\,(E\,)} = \frac{\frac{12}{51}}{\frac{22}{55}} = \frac{12}{22} = 0.54$

B) $P\,(F\,|Un\,) = \frac{P\,(F\,\cap Un\,)}{P\,(Un)} = \frac{\frac{15}{51}}{\frac{29}{55}} = \frac{15}{29} = 0.51$

C) $P\,(E\,|M\,) = \frac{P\,(E\,\cap M\,)}{P\,(M\,)} = \frac{\frac{12}{51}}{\frac{26}{55}} = \frac{12}{26} = 0.46$

▪**Bayes' Rule** : ⇨ $P(A) = \sum_{i=1}^{k} P\,(B_i \cap A\,) = \sum_{i=1}^{k} P\,(B_i)\,P\,(A|B_i)$

⇨$P\,(B_r|\,A\,) = \frac{P\,(B_r\,)\;P\,(A\,|B_r\,)}{\sum P\,(B_r\,)\,P\,(A\,|B_i\,)}$

▪**Example⑦**: three machines B_1 , B_2 , B_3 make 30% , 45% , 25% of the products . It is known from past that 2% , 2% , 3% of the products made by machine . What is the probability that it is defective ?

$P\,(B_1\,)\;P\,(A\,|B_1\,) = (\,0.3\,)(\,0.02\,) = 0.006$

$P\,(B_2\,)\;P\,(A\,|B_2\,) = (\,0.45\,)(\,0.02\,) = 0.0135$

$P\,(B_3\,)\;P\,(A\,|B_3\,) = (\,0.25\,)(\,0.03\,) = 0.005$

$P\,(A) = 0.006 + 0.0135 + 0.005 = 0.0245$

$$P\,(B_i|\,A\,) = \frac{P\,(B_i\,)\;P\,(A\,|B_i\,)}{P\,(A\,)} = \frac{0.3\;\times 0.02}{0.0245} = 0.244$$

Exercises

2.1 Page 38 : **A)** $S = \{ 8 , 16 , 24 , 40 , 48 \}$

B) $S = \{ x \mid x^2 + 4x - 5 = 0 \}$; $x^2 + 4x - 5 = 0 \therefore x = -5 , 1 \quad S = \{ -5 , 1 \}$

C) *when tossed one time* $S = \{ T , H \}$ *and we choise* T

when tossed two times $S = \{ TH , HT , HH , TT \}$ *and we choise* HT

when tossed one time $S = \{ THT , TTH, TTT, HHT , HTH , HHH \}$

and we choise HHH ; ***and the finale*** $\boldsymbol{S = \{ T , HT , HHH \}}$

E) $S = \{ x \mid 2x - 4 \geq 0 \; and \; x < 1 \}$;

$2x - 4 \geq 0 \; and \; 2x \geq 4 \; and \; x \geq 2 \quad ; S = \{ 2 , \ldots \ldots \ldots \ldots \infty \}$

if $x < 1$ *no number equal this rule* $\therefore S = \emptyset$

2.4 Page 38 :

A) $S = \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ , (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,4), (6,5), (6,6) \end{matrix} \right\}$

B) $S = \{ x, y \mid 1 \leq (x, y) < 6 \}$

2.8 Page 38 :

A) $S_1 = \{ (3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6) \}$

B) $S_2 = \{ (1,2), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2) \}$

C) $S_3 = \{ green , red \}$

$= \{ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

D) $S_1 \cap S_3 = \{ (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6) \}$

E) $S_1 \cap S_2 = \{ \emptyset \}$

f) $S_2 \cap S_3 = \{ (5,2), (6,2) \}$

2.14 Page 39 :

A) $A \cup C = \{0 , 2 , 3 , 4 , 5, 6 , 8 \}$

B) $A \cap B = \{ \emptyset \}$

C) $C' = \{ 0 , 1 , 6 , 7 , 8 , 9 \}$

D) $(C' \cap D) \cup B = \{ 1 , 6 , 7 \} \cup B = \{ 1 , 3 , 5 , 6 , 7 , 9 \}$

E) $(S \cap C)' = \{ 2 , 3 , 4 , 5 \}' = \{ 0 , 1 , 6 , 7 , 8 , 9 \}$

F) $A \cap C \cap D' = \{ 2 , 4 \}$

2.32 Page 47 :

A) $7\,P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7\times6\times5\times4\times3\times2\times1}{1} = 5040$

B) $(7 - 1)! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

2.40 Page 48 :

$$8\,P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 6720$$

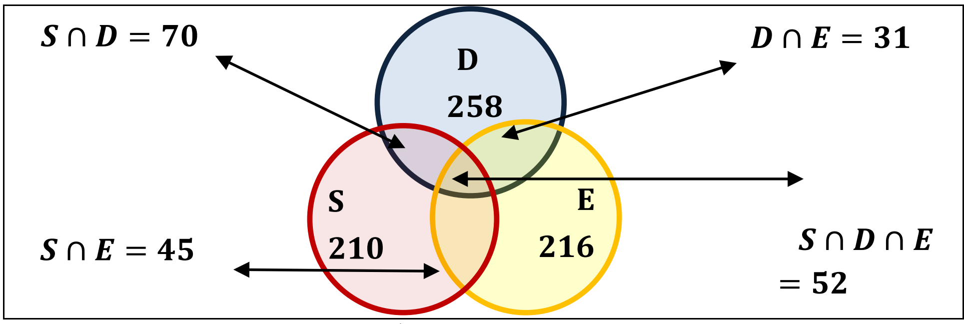
2.43 Page 48 :

$(5-1)! = 4! = 4 \times 3 \times 2 \times 1 = 24$

2.49 Page 48 :

$$8\,P_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3! \times 5!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1) \times (5 \times 4 \times 3 \times 2 \times 1)} = 56$$

2.54 Page 55 :



$\boldsymbol{S = 210 - 70 - 52 - 45 = 43}$

$\boldsymbol{D = 258 - 70 - 52 - 31 = 105}$

$\boldsymbol{E = 216 - 52 - 31 - 45 = 88}$

$\boldsymbol{Not\ do\ any\ activites = 500 - 434 = 66}$

A) $P(S \cap D') = \frac{43+45}{500} = \frac{88}{500} = 0.176 \ \ and\ P(S \cap E') = \frac{43+70}{500} = \frac{113}{500} = 0.226$

B) $P(E \cap S \cap D') = \frac{31}{500} = 0.062$

C) $P(S' \cap E') = \frac{105+66}{500} = \frac{71}{500} = 0.142$

2.55 Page 56 :

A : industrial in Shanghai , B : industrial in Beijing

$P(A) = 0.7$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

A) $P(A \cap B) = ??$; $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.7 + 0.4 - 0.8 = 0.3$

B) $P(A' \cap B') = ??$; $\because P(A) + P(A') = 1$

$\therefore P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

2.57 Page 56 :

A) $P(vowel\ letters) = \frac{5}{26} = 0.19$

B) $P(the\ letters\ before\ j) = \frac{9}{26} = 0.34$

C) $P(the\ letters\ after\ g) = \frac{19}{26} = 0.73$

2.78 Page 65 :

	Juniors	Seniors	Graduate students	Total
No in class	10	30	10	50
A	3	10	5	18

$$P(S|A) = \frac{P(S \cap A)}{P(A)} = \frac{\frac{10}{18}}{\frac{18}{18}} = \frac{5}{9} = 0.55$$

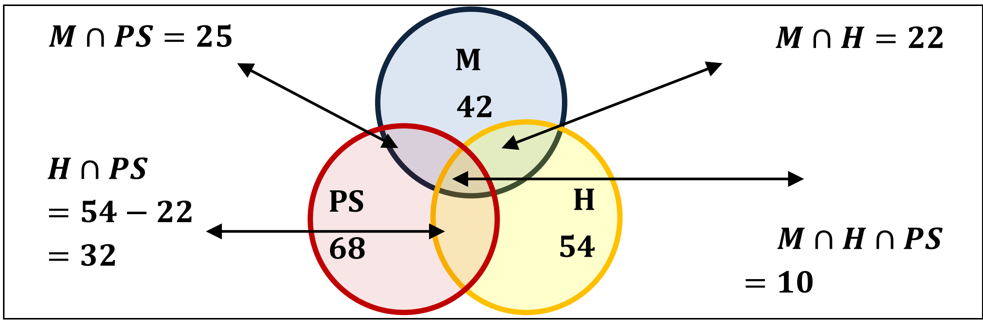
2.79 Page 65 :

	Elementary	Secondary	College	Total
Male	38	28	22	88
Female	45	50	17	112
Total	83	78	39	200

A) $P\,(\,M\,|S\,) = \frac{P\,(\,M\,\cap S\,)}{P\,(S\,)} = \frac{\frac{28}{200}}{\frac{78}{200}} = \frac{28}{78} = 0.35$

B) $P\,(\,C'\,|F\,) = \frac{P\,(\,\acute{C}\cap F)}{P\,(F)} = \frac{\frac{50+45}{200}}{112} = \frac{50+45}{112} = \frac{95}{112} = 0.84$

2.81 Page 65 :



***Not study any thing* = 8**
***Study all subjects* = 10**
***Study just history* = 7**

A) $P\,(M \cap H \cap PS\,) = = \frac{10}{68} = 0.147$

B) $P\,(M \cap H\,|PS'\,) = \frac{P\,(\,M\cap H\cap PS'\,)}{P\,(PS')} = \frac{22-10}{100-68} = \frac{12}{32} = 0.375$

2.101 Page 72 :

C : *an adult selscted has cancer* ; D : *the adult is diagnosed as having cancer*

$P\,(\,C\,) = 0.05$; $P\,(\,D|C\,) = 0.78$; $P\,(\,C'\,) = 1 - 0.05 = 0.95$; $P\,(\,D|C'\,) = 0.06$

$P\,(\,C \cap D\,) + P\,(\,C' \cap D\,) = (\,0.05\,)(\,0.78\,) + (\,0.95\,)(\,0.06\,) = 0.096$

2.105 Page 73 :

$P\,(\,jo) = 0.2$; $P\,(\,A\,|jo\,) = \frac{1}{200} = 0.005$

$P\,(\,t\,) = 0.60$; $P\,(\,A\,|t\,) = \frac{1}{100} = 0.01$

$P\,(\,j\,) = 0.15$; $P\,(\,A\,|j\,) = \frac{1}{90} = 0.011$

$P\,(\,j\,) = 0.05$; $P\,(\,A\,|j\,) = \frac{1}{200} = 0.005$

$P\,(A) = (0.005)(0.20) + (0.01)(0.60) + (0.15)(0.011) + (0.05)(0.005) = 8.9 \times 10^{-3}$

$P\,(\,jo|\,A\,) = \frac{(0.005)(0.20)}{8.9 \times 10^{-3}} = 0.1124$

Chapter 3

▪**Random variable** : is a function that associates a real number with each element in the sample space .

▪ The probability distribution of the discrete random variable X if , for each possible outcome x ,

1. $f(x) > 0$
2. $\sum_x f(x) = 1$
3. $P(X = x) = f(x)$

▪**Example①**: A shipment of 8 similar microcomputers to a retail outlet contains 3 that are defective . If a school makes a random purchase of 2 of these computers , find the probability distribution for the number of defectives .

$$f(0) = P(X = 0) = \frac{3\,C_0\ \ 5\,C_2}{8\,C_2} = \frac{10}{28} = 0.35$$

$$f(1) = P(X = 1) = \frac{3\,C_1\ \ 5\,C_1}{8\,C_2} = \frac{15}{28} = 0.53$$

$$f(2) = P(X = 2) = \frac{3\,C_2\ \ 5\,C_0}{8\,C_2} = \frac{3}{28} = 0.10$$

X	0	1	2
$f(x)$	0.35	0.53	0.10

Exercises

3.6 Page 88 :

A) $P\,(\,x\;>\;200\,)$

$$\int_{200}^{\infty} f\,(x)\;dx\;=\;\int_{200}^{\infty}\frac{2000}{(\,x+100)^{\,3}}\;dx\;=\;2000\int_{200}^{\infty}(\,x+100)^{-\,3}\;dx$$

$$=\;2000\left[\frac{(\,x+100)^{-\,2}}{-2}\right]_{200}^{\infty}\;=\;-1000\left[\frac{1}{(\,x+100)^{\,2}}\right]_{200}^{\infty}$$

$$=\;-1000\;\left[\frac{1}{(\,\infty+100)^{\,2}}-\frac{1}{(\,300)^{\,2}}\right]=\;-1000\;\left[0+\frac{1}{90000}\right]=\;\frac{1}{9}$$

B) $P\,(\,80\;<x\;<120\,)$

$$\int_{80}^{120} f\,(x)\;dx\;=\;\int_{80}^{120}\frac{2000}{(\,x+100)^{\,3}}\;dx\;=\;2000\int_{80}^{120}(\,x+100)^{-\,3}\;dx$$

$$=\;2000\left[\frac{(\,x+100)^{-\,2}}{-2}\right]_{80}^{120}\;=\;-1000\left[\frac{1}{(\,x+100)^{\,2}}\right]_{80}^{120}$$

$$=\;-1000\;\left[\frac{1}{(\,220)^{\,2}}-\frac{1}{(\,180)^{\,2}}\right]=\;-1000\,[1.02\;\times10^{-5}]=\;-\,0.102$$

3.7 Page 88 :

A) $P\,(\,x\;<1.2\,)$

$$\int_{0}^{1} x\;dx+\int_{1}^{1.2}(2-x)\;dx\;=\left[\frac{x^{\,2}}{2}\;\right]_{0}^{1}+\left[2x-\frac{x^{\,2}}{2}\right]_{1}^{1.2}$$

$$=\left[\frac{1}{2}-0\right]+\left[\;\left\{2(1.2)-\frac{1.2^{\,2}}{2}\right\}-\left\{2(1)-\frac{1^{\,2}}{2}\right\}\;\right]=0.68$$

B) $P\,(\,0.5\;<x\;<1\,)$

$$\int_{0.5}^{1} f\,(x)\;dx\;=\;\int_{0.5}^{1} x\;dx\;=\left[\frac{x^{\,2}}{2}\right]_{0.5}^{1}\;=\left[\frac{1^{\,2}}{2}-\frac{0.5^{\,2}}{2}\right]\;=0.375$$

3.9 Page 88 :

A) $P\,(\,0\;<x\;<1\,)$

$$\int_{0}^{1} f\,(x)\;dx\;=\;\frac{2}{5}\int_{0}^{1} x+2\;dx\;=\frac{2}{5}\left[\frac{x^{\,2}}{2}\;+2x\;\right]_{0}^{1}\;=\frac{2}{5}\left[\frac{1^{\,2}}{2}\;+2(1)-0\right]\;=\frac{2}{5}\;\left[\frac{5}{2}\right]=1$$

3.11 Page 38 :

$$f(\,0)=P\,(\,X\,=0\,)=\frac{2\;C_0\;\;5\;C_3}{7\;C_3}=\frac{2}{7}=0.28$$

$$f(\,1)=P\,(\,X\,=1\,)=\frac{2\;C_1\;\;5\;C_2}{7\;C_3}=\frac{4}{7}=0.57$$

$$f(\,2)=P\,(\,X\,=2\,)=\frac{2\;C_2\;\;5\;C_1}{7\;C_3}=\frac{1}{7}=0.14$$

X	0	1	2
$f(x)$	0.28	0.57	0.14

<u>3.12 Page 88</u> :

A) $P(T=5) = F(5) - F(4) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

B) $P(T > 3) = 1 - F(3) = 1 - \frac{1}{2} = \frac{1}{2}$

C) $P(1.4 < T < 6) = F(6) - F(1.4) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

<u>3.21 Page 89</u> :

A) $we\ put \int_0^1 f(x)\ dx = 1\ to\ find\ the\ value\ of\ k$

$$\int_0^1 k\sqrt{x}\ dx = 1 \ ⇨\ k\int_0^1 (x)^{\frac{1}{2}}\ dx = 1 \ ⇨\ k\left[\frac{2}{3}\ x^{\frac{3}{2}}\right]_0^1 = 1$$

$$⇨\ k\left[\frac{2}{3}\ (1)^{\frac{3}{2}} - 0\right] = 1 \ ⇨\ \frac{2}{3}k = 1 \ \therefore k = \frac{3}{2}$$

B) $P(0.3 < x < 0.6)$

$$\frac{3}{2}\int_{0.3}^{0.6} (x)^{\frac{1}{2}}\ dx = \frac{3}{2}\left[\frac{2}{3}\ x^{\frac{3}{2}}\right]_{0.3}^{0.6} = 1\left[(0.6)^{\frac{3}{2}} - (0.3)^{\frac{3}{2}}\right] = 0.30$$

<u>3.29 Page 90</u> :

A) $P(x > 200)$

$$\int_1^{\infty} f(x)\ dx = \int_1^{\infty} 3x^{-4}\ dx = 3\left[\frac{x^{-3}}{-3}\right]_1^{\infty} = -1\left[\frac{1}{x^3}\right]_1^{\infty} = -1\left[\frac{1}{\infty} - 1\right] = 1$$

<u>3.30 Page 89</u> :

A) $we\ put \int_{-1}^1 f(x)\ dx = 1\ to\ find\ the\ value\ of\ k$

$$\int_{-1}^1 k(3 - x^2)\ dx = 1 \ ⇨\ \int_{-1}^1 (3k - kx^2)\ dx = 1 \ ⇨\ \left[3k\,x - \frac{kx^3}{3}\right]_{-1}^1 = 1$$

$$⇨\ \left[3k\left(1-(-1)\right) - \frac{k}{3}\left(1^3 - (-1^3)\right)\right] = 1 \ ⇨\ 6k - \frac{2k}{3} = 1$$

$$⇨\ \frac{18k}{3} - \frac{2k}{3} = 1 \ ⇨\ \frac{16k}{3} = 1 \ \therefore k = \frac{3}{16} = 0.1875$$

Chapter 4

▪**Mean or excepted value** :Let X be a random variable with probability distribution $f\ (x\ )$ of X is :

▪**Variance** :Let X be a random variable with probability distribution $f\ (x\ )$ of X is

	Discrete	**Continuous**
μ_x	$E(x) = \sum_x x\, f\ (x)$	$E(x) = \int_{-\infty}^{\infty} x\, f(x)\ dx$
$\mu_{g(x)}$	$E\lvert g(x)\rvert = \sum_x g(x)\, f\ (x\ )$	$E\lvert g(x)\rvert = \int_{-\infty}^{\infty} g(x)\, f(x)\ dx$
$\sigma^2{}_x$	$E[(x-\mu)^2] = \sum_x (x-\mu)^2\, f\ (x\ )$	$E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\ dx$
$\sigma^2{}_{g(x)}$	$E[(g(x) - \mu_{g(x)})^2] = \sum_x (g(x) - \mu_{g(x)})^2\, f\ (x\ )$	$E[(g(x) - \mu_{g(x)})^2] = \int_{-\infty}^{\infty} (g(x) - \mu_{g(x)})^2\, f(x)\ dx$

▪**Example①**: A lot containing 7components is sampled by a quality inspector ; the lot contains 4 good components and 3 defective components . A sample of 3 is taken inspector. Find the excepted value of the number of good components ?

$$f(\ 0) = P\ (\ X = 0\ ) = \frac{3\ C_0\ \ 4\ C_3}{7\ C_3} = \frac{4}{35} = 0.11$$

$$f(\ 1) = P\ (\ X = 1\ ) = \frac{3\ C_1\ \ 4\ C_2}{7\ C_3} = \frac{18}{35} = 0.51$$

$$f(\ 2) = P\ (\ X = 2\ ) = \frac{3\ C_2\ \ 4\ C_1}{7\ C_3} = \frac{12}{35} = 0.34$$

$$f(\ 3) = P\ (\ X = 3\ ) = \frac{3\ C_3\ \ 4\ C_0}{7\ C_3} = \frac{1}{35} = 0.02$$

$$Mean\ \ \mu = E(x) \sum_x x\, f\ (x\ )$$

$$\mu = (0)\left(\frac{4}{35}\right) + \ (1)\left(\frac{18}{35}\right) + \ (2)\left(\frac{12}{35}\right) + \ (3)\left(\frac{1}{35}\right) = \frac{9}{7} = 1.2$$

X	0	1	2	3
$f(\ x)$	0.11	0.51	0.34	0.02

▪**Example②**: from this table, Find **A)** μ_x , **B)** $\sigma^2{}_x$, **C)** $\sigma^2{}_{g(x)} if\ g(x) = 2x + 1$

X	0	1	2
$f(\ x)$	0.2	0.4	0.4

A) $Mean\ \ \mu = E(x) \sum_x x\, f\ (x\ )$

$\mu = (0)(\ 0.2\ ) + \ (1)(\ 0.4\ ) + \ (2)(\ 0.4\ ) = 1.2$

B) $E[(x-\mu)^2] = \sum_x (x-\mu)^2\, f\ (x\ )$

$\sigma^2{}_x = (1.44)(\ 0.2\ ) + \ (0.04)(\ 0.4\ ) + \ (0.64)(\ 0.4\ ) = 0.56$

C) $\mu_{g(x)} = \sum_x g(x)\, f\ (x\ ) = (1)(\ 0.2\ ) + \ (3)(\ 0.4\ ) + \ (5)(\ 0.4\ ) = 3.4$

$E[(g(x) - \mu_{g(x)})^2] = \sum_x (g(x) - \mu_{g(x)})^2\, f\ (x\ )$

$\sigma^2{}_{g(x)} = (5.76)(\ 0.2\ ) + \ (0.16)(\ 0.4\ ) + \ (2.56)(\ 0.4\ ) = 2.24$

x	$x-\mu$	$(x-\mu)^2$	$g(x)$	$g(x)-\mu_{g(x)}$	$\left(g(x)-\mu_{g(x)}\right)^2$
0	-1.2	1.44	1	-2.4	5.76
1	-0.2	0.04	3	-0.4	0.16
2	0.8	0.64	5	-1.6	2.56

▪**Example③**: $f(x) = \begin{cases} \frac{x^2}{3} \; ; -1 < x < 2 \\ 0\,, otherwise \end{cases}$

Find **A)** μ_x , **B)** $\sigma^2{}_x$, **C)** $\mu_{g(x)}$, **D)** $\sigma^2{}_{g(x)} if\ g(x) = 4x+3$

A) $Mean\ \mu = \int_{-\infty}^{\infty} x\, f(x)\ dx$

$$= \int_{-1}^{2} x \left(\frac{x^2}{3}\right) dx = \frac{1}{3}\int_{-1}^{2} x^3\ dx = \frac{1}{3}\left[\frac{x^4}{4}\right]_{-1}^{2} = \frac{1}{12}\,[2^4 - (-1)^4] = \frac{15}{12} = 1.25$$

B) $\sigma^2{}_x = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\ dx$

$$= \int_{-1}^{2} (x-1.25)^2 \left(\frac{x^2}{3}\right) dx = \frac{1}{3}\int_{-1}^{2} (\,x^2 - 2.5x + 1.56\,)\, x^2\ dx$$

$$= \frac{1}{3}\int_{-1}^{2} x^4 - 2.5x^3 + 1.56x^2\ dx = \frac{1}{3}\left[\frac{x^5}{5} - 0.625x^4 + 0.52x^3\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[\left\{\left(\frac{2^5}{5}\right) - 0.625(2)^4 + 0.56(2)^3\right\} - \left\{\left(\frac{1^5}{5}\right) - 0.625(1)^4 + 0.56(1)^3\right\}\ \right]$$

$$= \frac{1}{3}\left[\frac{22}{25} - \frac{27}{200}\right] = \frac{1}{3}\left[\frac{149}{200}\right] = 0.248$$

C) $\mu_{g(x)} = \int_{-\infty}^{\infty} g(x)\, f(x)\ dx$

$$= \int_{-1}^{2} (4x+3) \left(\frac{x^2}{3}\right) dx = \frac{1}{3}\int_{-1}^{2} 4x^3 + 3x^2\ dx = \frac{1}{3}[x^4 + x^3]_{-1}^{2}$$

$$= \frac{1}{3}\,[\{2^4 + 2^3\} - \{(-1)^4 + (-1)^3\}] = \frac{1}{3}[24 - 0] = 8$$

D) $\sigma^2{}_{g(x)} = \int_{-\infty}^{\infty} (g(x) - \mu_{g(x)})^2\, f(x)\ dx$

$$= \int_{-1}^{2} (4x+3-8)^2 \left(\frac{x^2}{3}\right) dx = \int_{-1}^{2} (4x+5)^2 \left(\frac{x^2}{3}\right) dx = \int_{-1}^{2} 16x^2 - 40x + 25 \left(\frac{x^2}{3}\right) dx$$

$$= \frac{1}{3}\int_{-1}^{2} 16x^4 - 40x^3 + 25x^2\ dx = \frac{1}{3}\left[\frac{16}{5}x^5 - 10x^4 + \frac{25}{3}x^3\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[\left\{\frac{16}{5}(2)^5 - 10(2)^4 + \frac{25}{3}(2)^3\right\} - \left\{\frac{16}{5}(-1)^5 - 10(-1)^4 + \frac{25}{3}(-1)^3\right\}\ \right]$$

$= \frac{1}{3}\,[\{102.4 - 160 + 66.67\} - \{-3.2 + 10 - 8.33\}\,] = = \frac{1}{3}\,[9.07 + 1.53\,] = \frac{1}{3}[10.6] =$ 3.53

▪**Example④**: from this table, Find $E(x-1)^2$?

X	0	1	2
$f(\,x)$	0.2	0.3	0.5

$E(ax+b) = aE(x) + E(x)\,;\ E(ax^2+bx+c) = aE(x^2) - bE(x) + E(c)$

⇨$E(x) = \sum x\, f(x) \;\; = (0)(\,0.2\,) + \;(1)(\,0.3\,) + \;(2)(\,0.5\,) = 1.3$

⇨$E(x^2) = \sum x^2 f(x) = (0^2)(0.2) + (1^2)(0.3) + (2^2)(0.5) = 2.3$

⇨$E(x-1)^2 = E(x^2) - 2E(x) + E(1) = 2.3 + 2(1.3) + 1 = 0.7$

▪**Example⑤**: $f(x) = \begin{Bmatrix} 6x(1-x) \ ; \ 0 < x < 1 \\ 0 \, , otherwise \end{Bmatrix} \ Find \ E(x-1)^2 \ ?$

⇨$E(x) = \int_{-\infty}^{\infty} x \, f(x) \, dx$

$$= \int_0^1 x \, 6x(x-1) \, dx = 6\int_0^1 x^2 - x^3 \, dx = 6\left[\frac{x^3}{3} - \frac{x^4}{3}\right]_0^1 = 6\left[\frac{1^3}{3} - \frac{1^4}{4} - 0\right] = 6\left[\frac{1}{12}\right] = \frac{1}{2}$$

⇨$E(x^2) = \int_{-\infty}^{\infty} x^2 \, f(x) \, dx$

$$= \int_0^1 x^2 6x(x-1) \, dx = 6\int_0^1 x^3 - x^4 \, dx = 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 6\left[\frac{1^3}{4} - \frac{1^4}{5} - 0\right] = 6\left[\frac{1}{20}\right]$$

$$= \frac{3}{10}$$

⇨$E(x-1)^2 = E(x^2) - 2E(x) + E(1) = 0.3 + 2(0.5) + 1 = 0.3$

▪**Example⑥**: from this table, Find $E(xy)$?

X	0	1	2
Y	1	2	3
$f(x)$	0.2	0.4	0.4
$f(y)$	0.1	0.5	0.4

⇨$E(x) = \sum x \, f(x) = (0)(0.2) + (1)(0.4) + (2)(0.4) = 1.2$

⇨$E(y) = \sum y \, f(y) = (1)(0.1) + (2)(0.5) + (3)(0.4) = 2.3$

⇨$E(xy) = E(x)E(y) = 1.2 \times 2.3 = 2.76$

▪**Example⑦**: $f(x) = \begin{Bmatrix} \frac{x(1+3y^2)}{4} \ ; \ 0 < x < 2 \ ; \ 0 < y < 1 \\ 0 \, , otherwise \end{Bmatrix} \ Find \ E(xy) \ ?$

$$E(x) = \iint_{-\infty}^{\infty} x \, f(x,y) \, dx \, dy = \int_0^1 \int_0^2 \frac{x^2(1+3y^2)}{4} \, dx \, dy$$

$$= \int_0^1 \left[\frac{x^3(1+3y^2)}{12}\right]_0^2 dy = \int_0^1 \frac{2}{3}(1+3y^2) \, dy = \frac{2}{3}[y + y^3]_0^1 = \frac{2}{3}[1 + 1^3 + 0] = \frac{4}{3}$$

$$E(y) = \iint_{-\infty}^{\infty} y \, f(x,y) \, dx \, dy = \int_0^1 \int_0^2 \frac{xy(1+3y^2)}{4} \, dx \, dy$$

$$= \int_0^1 \left[\frac{x^2 y(1+3y^2)}{8}\right]_0^2 dy = \int_0^1 \left[\frac{y(1+3y^2)}{2}\right] dy = \frac{1}{2}\int_0^1 [y + 3y^3] \, dy$$

$$= \frac{1}{2}\left[\frac{y^2}{2} + \frac{3y^3}{4}\right]_0^1 = \frac{1}{2}\left[\frac{1^2}{2} + \frac{3(1)^3}{4} + 0\right] = \frac{1}{2}\left[\frac{5}{4}\right] = \frac{5}{8} = 0.625$$

$$E(xy) = E(x) \, E(y) = \left(\frac{4}{3}\right)\left(\frac{5}{8}\right) = \frac{5}{6} = 0.833$$

Exercises

4.2 Page 113 :

X	0	1	2	3
$f(x)$	0.42	0.42	0.14	0.01

$$\mu = (0)(0.42) + (1)(0.42) + (2)(0.14) + (3)(0.01) = 0.73$$

4.12 Page 113 & 4.50 Page 122 :

$$\mu = \int_0^1 x\,2(1-x)\ dx = 2\int_0^1 x - x^2\ dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1^2}{2} - \frac{1^3}{3} - 0\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}$$

$$\sigma^2{}_x = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx = \int_0^1 \left(x - \frac{1}{3}\right)^2 2(1-x)\ dx$$

$$= 2\int_0^1 \left(x^2 - \frac{2}{3}x + \frac{1}{9}\right)(1-x)\ dx = 2\int_0^1 \left(-x^3 + \frac{5}{3}x^2 - \frac{5}{9}x\right) dx$$

$$= 2\left[-\frac{x^4}{4} + \frac{5x^3}{9} - \frac{5x^2}{18}\right]_0^1 = 2\left[-\frac{1^4}{4} + \frac{5^3}{9} - \frac{5^2}{18} - 0\right] = 2\left[\frac{49}{4}\right] = \frac{49}{2} = 24.5$$

4.17 Page 114 & 4.41 Page 122 :

X	-3	6	9
$f(x)$	$\frac{1}{6} = 0.167$	$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.33$
$g(x)$	$(-5)^2 = 25$	$13^2 = 169$	$19^2 = 361$
$g(x) - \mu_{g(x)}$	-182.8	-40	152
$(g(x) - \mu_{g(x)})^2$	33415.84	1600	23104

$$\mu_{g(x)} = (0.167)(25) + (0.5)(169) + (0.33)(361) = 207.8$$

$$\sigma^2{}_{g(x)} = (0.167)(33415.84) + (0.5)(1600) + (0.33)(23104) = 14004.76$$

4.18 Page 114 :

X	0	1	2	3
$f(x)$	0.42	0.42	0.14	0.01

$$\mu = (0)(0.42) + (1)(0.42) + (2)(0.14) + (3)(0.01) = 0.73$$

4.20 Page 114 :

$$\mu_{g(x)} = E\left(e^{\frac{2x}{3}}\right) = \int_0^{\infty} x\left(e^{\frac{2x}{3}}\right)(e^{-x})\ dx = \int_0^{\infty}\left(e^{\frac{-x}{3}}\right) = \left[\left(e^{\frac{-x}{3}}\right)\right]_0^{\infty} = 3$$

4.32 Page 115 :

X	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

B) $E(x) = \mu = (0)(0.41) + (1)(0.37) + (2)(0.16) + (3)(0.05) + (4)(0.01) =$ 0.88

C) $E(x^2) = \sum x^2 f(x) = (0^2)(0.41) + (1^2)(0.37) + (2^2)(0.16)$
$+ (3^2)(0.05) + (4^2)(0.01) = 1.62$

4.35 Page 122 :

X	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	0.3	0.04
$x-\mu$	- 2.11	- 1.11	- 0.11	0.89	1.89
$(x-\mu)^2$	4.4521	1.2321	0.0121	0.7921	3.5721

. $\mu = (2)(0.01) + (3)(0.25) + (4)(0.4) + (5)(0.3) + (6)(0.04) = 4.11$

$$\sigma^2{}_x = (4.4521)(0.01) + (1.2321)(0.25) + (0.0121)(0.4) + (0.7921)(0.3) + (3.5721)(0.04) = 0.7379$$

4.56 Page 134 :

⇨$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_0^1 x^2\, dx + \int_1^2 2x - x^2\, dx = 1$$

⇨$E(x^2) = \int_{-\infty}^{\infty} x^2\, f(x)\, dx$

$$= \int_0^1 2x^3\, dx + \int_1^2 2x - x^2\, dx = \frac{7}{6} = 1.167$$

⇨$E(Y) = 60E(x^2) + 39E(x) = 60(1.167) + 39(1) = 109$

4.58 Page 134 :

⇨$E(x) = (2)(0.40) + (4)(0.60) = 3.20$

⇨$E(y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3$

⇨$E(2x - 3y) = 2E(x) - 3E(y) = 2(3.20) - 3(3) = -2.6$

⇨$E(xy) = 3.20 \times 3 = 9.6$

4.70 Page 135 :

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_2^{\infty} x\left(\frac{8}{x^3}\right) dx = 8\int_2^{\infty} x^{-2}\, dx = 8\left[\frac{x^{-1}}{-1}\right]_2^{\infty}$$

$$= -8\left[\frac{1}{x}\right]_2^{\infty} = -8\left[\frac{1}{\infty} - \frac{1}{2}\right] = -8\left[0 - \frac{1}{2}\right] = 4$$

$$E(y) = \int_{-\infty}^{\infty} y\, f(y)\, dy = \int_0^1 y(2y)\, dx = 2\int_0^1 y^2\, dx = 2\left[\frac{y^3}{3}\right]_0^1 = \frac{2}{3}[1-0] = \frac{2}{3} = 0.67$$

$E(xy) = E(x)\, E(y) = (4)(0.67) = 2.67$

4.87 Page 137 :

A) $E(x) = \frac{1}{900}\int_0^{\infty} x\left(e^{-\frac{x}{900}}\right) dx = 900$

B) $E(x^2) = \frac{1}{900}\int_0^{\infty} x^2\left(e^{-\frac{x}{900}}\right) dx = 1620000$

C) $E(x^2) - [E(X)]^{2} = 1620000 - 900^2 = 810000$ $and\ \sigma = 900$

Chapter 5

▪**Binomial Distribution** : A Bernoulli trial can result in a success with probability p and a failure :

$$b\ (x\ ; n\ , p\ ) = \binom{n}{x}\ p^x\ q^{n-x}\ \ ,\ \ x = 0\ , 1\ , 2\ , \ldots\ldots\ldots , n$$

x : $Random\ variable$; p : $number\ of\ success\ (1-q)$;
n : $number\ of\ trails$; q : $number\ of\ faliure\ (1-p)$;

▪**Mean** : $\mu = np$

▪ **Variance** : $\sigma^2 = npq$

▪**Example①**: $Given\ n = 3\ , p = \frac{1}{4}$; Find the binomial distribution when x = 0 , 1 , 2, 3

$q = 1 - p = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$for\ x = 0\ ;\ \ b\left(0\ ; 3\ , \dfrac{1}{4}\right) = 3C_0\ \left(\dfrac{1}{4}\right)^0\ \left(\dfrac{3}{4}\right)^{3-0=3}\ \ = 0.42$

$for\ x = 1\ ;\ \ b\left(1\ ; 3\ , \dfrac{1}{4}\right) = 3C_1\ \left(\dfrac{1}{4}\right)^1\ \left(\dfrac{3}{4}\right)^{3-1=2}\ \ = 0.42$

$for\ x = 2\ ;\ \ b\left(2\ ; 3\ , \dfrac{1}{4}\right) = 3C_2\ \left(\dfrac{1}{4}\right)^2\ \left(\dfrac{3}{4}\right)^{3-2=1}\ \ = 0.14$

$for\ x = 3\ ;\ \ b\left(3\ ; 3\ , \dfrac{1}{4}\right) = 3C_3\ \left(\dfrac{1}{4}\right)^3\ \left(\dfrac{3}{4}\right)^{3-3=0}\ \ = 0.01$

X	0	1	2	3
$b(x)$	0.42	0.42	0.14	0.01

▪**Example②**:The probability that a patient recovers from a rare blood disease is 0.4 . If 15 people are known to have contracted this disease , what is the probability that A) at least 10 survive , B) from 3 to 8 survive , C) exactly 5 survive ?

A) $(p\ \geq 10\ )$

$1 - p(x\ < 10\ )\ \ \therefore 1 - \sum_{x=0}^{9} b\ (0\ ; 15\ , 0.4\ )$ - from the table page 745 -

$= 1 - 0.9662 = 0.0338$

B) $p(3\ \leq x\ < 8\ )$

$\therefore \sum_{x=0}^{8} b\ (x\ ; 15\ , 0.4\ ) - \sum_{x=0}^{2} b\ (x\ ; 15\ , 0.4\ ) -$ - from the table page 745 -

$= 0.9050 - 0.0271 = 0.8779$

C) $p(X = 5\ )$

$\therefore \sum_{x=0}^{5} b\ (x\ ; 15\ , 0.4\ ) - \sum_{x=0}^{4} b\ (x\ ; 15\ , 0.4\ ) -$ - from the table page 745 -

$= 0.4032 - 0.2173 = 0.2173$

▪**Poisson Distribution** :

$$P\ (\ x\ ;\lambda t\ ) = \frac{e^{-\lambda t}\ (\lambda t)^x}{x\ !}\ \ ,\ \ x = 0\ ,1\ ,2\ ,\dots\dots\dots.,n$$

▪**Poisson Probability** :

$$P\ (\ r\ ;\lambda t\ ) = \sum_{x=0}^{r} P\ (\ x\ ;\lambda t\ )$$

x : $Random\ variable$; λ : $number\ of\ outcomes$; t : $time\ interval$

▪ **Mean** : $\mu = \lambda t$ ▪ **Variance** : $\sigma^2 = npq$

▪**Example③**:During a laboratory experiment the average number of radioactive particles passing through a counter in 1 millisecond is 4 . What is probability that 6 particles enter the counter in given milliseconds ?

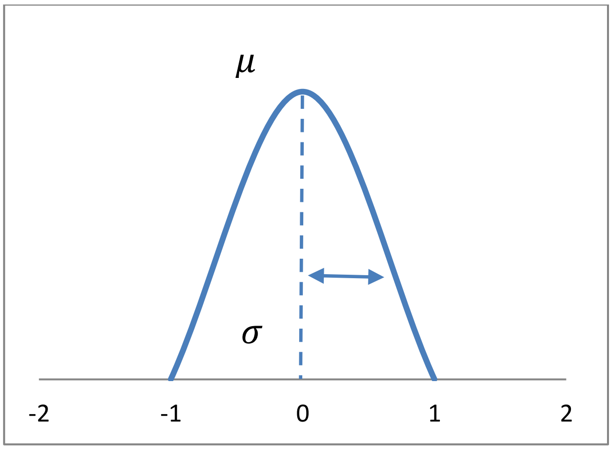
$x = 6$; $\lambda t = 4\ \times 1 = 4$

$$P\ (\ 6\ ;4\ ) = \frac{e^{-4}\ (4)^6}{6\ !} = \sum_{x=0}^{6} P\ (\ x\ ;4\ ) - \sum_{x=0}^{5} P\ (\ x\ ;4\ )$$

$= 0.8893 - 0.7851 = 0.0142$ - from the table page 748 –

Chapter 6

▪Standard Normal Distribution :



$\mu = 1 \; ; \; \sigma = 0 \; ;$

$$Z = \frac{x - \mu}{\sigma}$$

$Z : normal \; Area$

$Total \; Area = 1$

Example①: $-$ *Back to example* 6.2 *page* 178 $-$

A) $Total \; Area = 1 \; ; \; Z = 1.84$ *from the table at page* 752 *we found* $= 0.9671$

$\therefore 1 - 0.9671 = \; 0.0329$ *to right*

B) $P(-1.97 \; < Z < 0.86\,)$ *from the table at page* 752

$\therefore Z_{0.86} - \; Z_{1.97} = 0.8051 - 0.0244 = 0.7807$

Exercises

6.1 Page 185 :

A) $from\ the\ table\ at\ page\ 752\ we\ found\ \ Z_{0.86} = 0.9236\ \ to\ the\ left$

B) $1 - \ Z_{-0.89} = 1 - 0.1867 = 0.8133\ to\ the\ right$

C) $Z_{-2.16} - \ Z_{-0.65} = 0.0154 - 0.2578 = -0.2424$

D) $Z_{-1.39} = 0.0823\ \ to\ the\ left$

E) $1 - \ Z_{-1.96} = 1 - 0.9761 = 0.0239\ \ to\ the\ right$

C) $Z_{-0.48} - \ Z_{1.74} = 0.3156 - 0.9591 = -0.6435$

6.2 Page 185 :

A) $Z_k = 0.3622\ \ \ to\ the\ right\ \ \therefore k = 0.35$

B) $1 - Z_k = 1 - 0.1131 = 0.8869\ \ to\ the\ left\ \ \therefore k = 1.21$

6.4 Page 186 :

A) $\mu = 30\ ;\ \sigma = 6\ ; x = 17\ ;\ \ Z = \frac{x - \mu}{\sigma} = \frac{17-30}{6} = \ -\frac{13}{6} = \ -2.16$

$$1 - \ Z_{-2.16} = 1 - 0.0154 = 0.9846\ to\ the\ right$$

B) $\mu = 30\ ;\ \sigma = 6\ ; x = 22\ ;\ \ Z = \frac{x - \mu}{\sigma} = \frac{22-30}{6} = \ -\frac{4}{3} = \ -1.33$

$$Z_{-1.33} = 0.0918\ \ to\ the\ left$$

C) $\mu = 30\ ;\ \sigma = 6\ ; x_1 = 32\ ;\ x_2 = 41$

$Z_1 = \frac{x - \mu}{\sigma} = \frac{32-30}{6} = \ \frac{1}{3} = \ 0.33\ ;\ \ Z_2 = \frac{x - \mu}{\sigma} = \frac{41-30}{6} = \ \frac{11}{6} = \ 1.83$

$Z_{0.33} - \ Z_{1.83} = 0.9664 - 0.6293 = 0.3371$

6.5 Page 186 :

A) $\mu = 18\ ;\ \sigma = 2.5\ ; x = 15\ ;\ \ Z = \frac{x - \mu}{\sigma} = \frac{15-18}{6} = \ -\frac{3}{6} = -\frac{1}{2} = \ -0.5$

$$Z_{-0.5} = 0.1151\ \ to\ the\ left$$

B) $1 - Z_k = 1 - 0.2236\ \ = \ 0.0.76\ \ to\ the\ left\ \ \therefore k = \ -2.43$

$\because\ \ Z_k = \frac{x - \mu}{\sigma} = -2.43\ \ ;\ \ \ \therefore x = \ Z_k\sigma + \ \mu = \ -2.43\ \times 2.5 + 18 = \ 11.925\ \ \approx 12$

C) $Z_k = 0.1814\ \ \ to\ the\ right\ \ \therefore k = \ -0.9$

$\because\ \ Z_k = \frac{x - \mu}{\sigma} = -0.91\ \ ;\ \ \ \therefore x = \ Z_k\sigma + \ \mu = \ -0.9\ \times 2.5 + 18 = \ 15.75$

D) $\mu = 18\ ;\ \sigma = 2.5\ ; x_1 = 17\ ;\ x_2 = 21$

$Z_1 = \frac{x - \mu}{\sigma} = \frac{17-18}{2.5} = \ -\frac{2}{5} = -\ 0.4\ ;\ \ Z_2 = \frac{x - \mu}{\sigma} = \frac{21-18}{2.5} = \ -\frac{6}{5} = \ 1.2$

$Z_{1.2} - Z_{-0.4} = 0.8849 - 0.3446 = 0.5403$

Chapter 8

Sample Mean	Sample Variance	Standard deviation
$\bar{x} = \sum \frac{xi}{n}$	$S^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{\sum x^2 - \sum \frac{x^2}{n}}{n-1}$	$S^2 = \sqrt{S}$

▪**Poisson Distribution** : if $\bar{x}$ is the mean of a random sample size , n taken from a population with mean μ and the finite variance σ^2 .

$$Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \quad , \quad Z = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\left(\frac{\sigma^2{}_1}{n_1}\right)+\left(\frac{\sigma^2{}_2}{n_2}\right)}}$$

$$\sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} \quad , \quad \mu_{\bar{x}} = \mu$$

▪**Test Statistic** : χ^2 chi-square , S Sample Variance , n Sample Size , σ^2 Population Variance .

$$\chi^2 = \frac{S^2(n-1)}{\sigma^2}$$

▪**Example①**: An electrical firm manufactures light bulbs have a length of life , with mean =equal to 800 and a standard deviation of 40 hours . Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours .

$n = 16$, $\mu = 800$, $\sigma = 40$, $P(\bar{x} < 775) = ??$

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{775-800}{\frac{40}{\sqrt{16}}} = -\frac{25}{10} = -2.5$$

$\therefore P(\bar{x} < 775) = P(z < -2.5)$ *from the table at page* 751 *we found*

$= 0.0062$

$\therefore percentage = 0.0062 \times 100 = 0.6\ \%$

▪**Example②**: Find the statistic for these data 1.9 , 2.4 , 3 , 3.5 , 4.2 standard deviation is 1 . $n = 5$, $\sigma = 1$, $\sigma^2 = 1$,

$$\bar{x} = \frac{1.9+2.4+3+3.5+4.2}{5} = 3$$

x	$x-\bar{x}$	$(x-\bar{x})^2$	$\bar{x} = 3$
1.9	-1.1	1.21	$\sum(x-\bar{x})^2 = 3.26$
2.4	-0.6	0.36	
3	0	0	$S^2 = \frac{3.26}{4} = 0.815$
3.5	0.5	0.25	
4.2	1.2	1.44	$S = \sqrt{0.815} = 0.903$

$$\chi^2 = \frac{0.815^2 \times (5-1)}{1} = 2.6569$$

▪ **t − Distribution** :

$if \;\; n < 30 \; we \; use \; this \; rule \; t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)} \qquad , if \;\; n > 30 \; we \; use \; this \; rule \; t = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

⇨ $Degrre \; fredom : \; \Upsilon = n - 1$

Exercises

8.3 Page 234 :

A) $\bar{x} = \frac{2+1+3+0+1+3+6+0+3+3+5+2+1+4+2}{15} = 2.4$

B) $Median = 2$ *we must arrange the numbers from small to big number*

C) $Mode = 3$

8.4 Page 234 :

A) $\bar{x} = \frac{5+11+9+5+10+15+6+10+5+10}{10} = 8.6$

B) $Median = 9.5$

C) $Mode = 5$ *and* 10

8.7 Page 235 :

A) $\bar{x} = \frac{100+40+75+15+20+100+75+50+30+10+55+75+25+50+90+80+15+25+45+100}{20} = 53.75$

B) $Mode = 75$ *and* 100

8.8 Page 235 :

$\bar{x} = 22.2$ *days and* $Median = 14$ *days and* $Mode = 8$ *days* .

$\bar{x}$ is the best measure of the center of the data. The mean should not be used on account of the extreme value 95 , and the mode is not desirable because the sample size is too small.

8.15 Page 235 :

A) $S^2 = \frac{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)\, 2}{n\,(n-1)} = \frac{(6)(270)-(33)^2}{(6)(5)} = 5.1$

Multiplying each observation by 3 gives $S^2 = 9 \times 5.1 = 45.9$

B) *Adding 5 to each observation does not change the variance. Hence* $S^2 = 51.$

8.19 Page 252 :

A) ① $for\ n\ 64: \ \sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} = \frac{56^2}{\sqrt{64}} = 3.29$

② $for\ n\ 169: \ \sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} = \frac{56^2}{\sqrt{196}} = 2.24$

Therefore, the variance of the sample mean is reduced from 3.29 to 2.24 when the sample size is increased from 64 to 196.

B) ① $for\ n\ 784: \ \sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} = \frac{56^2}{\sqrt{7.84}} = 1.12$

② $for\ n\ 49: \ \sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} = \frac{56^2}{\sqrt{49}} = 4.48$

Therefore, the variance of the sample mean is increased from 1.12 to 4.48 when the sample size is decreased from 784 to 49.

8.23 Page 252 :

X	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1
$x - \mu$	-1.31	-0.31	0.59	1.69
$(x - \mu)^2$	1.7161	0.0961	0.4761	2.8561

A) $Mean\ \mu = E(x)\ \sum x\ f\ (x\ )$

$\mu = (4)(\ 0.2) + \ (5)(\ 0.4) + \ (6)(\ 0.3) + \ (7)(\ 0.1) = 5.31$

$\sigma^2{}_x = (1.7161)(\ 0.2) + \ (0.0961)(\ 0.4) + \ (0.4761)(\ 0.3) + \ (2.8561)(\ 0.1) = 0.81$

B) $When\ n = \mathbf{36}\ ;\ \mu_{\bar{x}} = \mu = 5.31\ and\ \sigma^2{}_{\bar{x}} = \frac{\sigma^2}{\sqrt{n}} = \frac{0.81}{6} = 0.0225$

C) $Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{5.5-5.3}{\frac{0.9}{\sqrt{36}}} = 1.33\ , P(\bar{x} < 5.5) = P(Z < 1.33) = 0.9082$

8.28 Page 252 :

$Given:\ n_1 = 25\ \ , n_2 = 36\ , \mu_1 = 80\ , \mu_2 = 75\ , \sigma_1 = 5\ , \sigma_2 = 3$

① $P(3.4 < \ \overline{x_1} - \overline{x_2} < 5.9\ ) \ = ??\ \ ;\ \ \ \overline{x_1} - \overline{x_2} \ = 3.4$

$$Z = \frac{(3.4) - (80 - 75)}{\sqrt{\left(\frac{5^2}{25}\right) + \left(\frac{3^2}{36}\right)}} = \ -1.43$$

② $P(3.4 < \ \overline{x_1} - \overline{x_2} < 5.9\ ) \ = ??\ \ ;\ \ \ \overline{x_1} - \overline{x_2} \ = 5.9$

$$Z = \frac{(5.9) - (80 - 75)}{\sqrt{\left(\frac{5^2}{25}\right) + \left(\frac{3^2}{36}\right)}} = \ 0.8$$

$\therefore P(-1.43 < Z < 0.8) = \ Z_{0.8} - \ Z_{-1.43} = 0.7881 - 0.0764 = 0.7117$

$\therefore Percentage = 0.7117\ \times 100 = 71.71$

8.50 Page 265 :

$Given:\ t_{0.025} = 2.31\ \ with\ \Upsilon = 15\ \ \ , \bar{x} = 27.5\ , S = 5\ , n = 16, \mu = 30$

$\because\ \ n < 30\ we\ use\ this\ rule\ t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)} = \ t = \frac{27.5 - 30}{\left(\frac{5}{\sqrt{16}}\right)} = \ -2$

Therefore, The answer is -2 is include in period from -2.131 to 2.131 and we conclude from this ; the batteries will work average 30 hours because the claim of the company is true .

8.52 Page 265 :

$$\bar{x} = \frac{0.6 + 0.7 + 0.3 + 0.4 + 0.5 + 0.4 + 0.2 + 0.7}{8} = 0.475$$

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\bar{x} = 0.475$
0.6	0.125	0.0156	$\sum(x - \bar{x})^2 = 0.234$ $S^2 = \frac{0.234}{7} = 0.03$
0.7	0.225	0.0506	
0.3	-0.175	0.0306	
0.4	-0.075	0.00562	
0.5	0.025	0.000625	
0.4	-0.075	0.00562	$S = \sqrt{0.03} = 0.1832$
0.2	-0.275	0.0756	
0.7	0.225	0.0506	

$\because\ \ n < 30\ we\ use\ this\ rule\ t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)} = \ t = \frac{0.475 - 0.5}{\left(\frac{0.1832}{\sqrt{8}}\right)} = \ -0.386$

Chapter 9

▪Single Sample : Estimating the Mean :

$H_O : \mu = 30$	⇨	$H_A : \mu \neq 30$
$H_O : \mu \leq 30$	⇨	$H_A : \mu > 30$ Right tail test
$H_O : \mu \geq 30$	⇨	$H_A : \mu < 30$ left tail test

▪Confidence Interval of μ and σ is Known :

$$\bar{x} - Z_{\left(\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\left(\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}} \qquad \text{we use this rule if } n \geq 30$$

▪*Error* (∈) : $Z_{\left(\frac{\alpha}{2}\right)} \frac{\sigma}{\sqrt{n}}$

▪*Sample Size* (*n*) : $\left(\frac{Z_{\left(\frac{\alpha}{2}\right)} \sigma}{\epsilon}\right)^2$

▪Confidence Interval of μ and σ is Unknown :

$$\bar{x} - t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}} \qquad \text{we use this rule if } n < 30$$

▪Single Sample : Estimating the Mean the difference between means :

Case ① : $\sigma^2{}_1$ and $\sigma^2{}_2$ are Known

$$(\overline{x_1} - \overline{x_2}) - Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{\sigma^2{}_1}{n_1}\right) + \left(\frac{\sigma^2{}_2}{n_2}\right)} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{\sigma^2{}_1}{n_1}\right) + \left(\frac{\sigma^2{}_2}{n_2}\right)}$$

Case ② : $\sigma^2{}_1$ and $\sigma^2{}_2$ are UnKnowns and $\sigma^2{}_1 = \sigma^2{}_2$

$$(\overline{x_1} - \overline{x_2}) - t_{\left(\frac{\alpha}{2}\right)} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + t_{\left(\frac{\alpha}{2}\right)} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\text{Degrre fredom} : \Upsilon = n_1 + n_2 - 2$$

$$S_P = \sqrt{\frac{(n_1 - 1)S^2{}_1 + (n_2 - 1)S^2{}_2}{n_1 + n_2 - 2}}$$

Case ③ : $\sigma^2{}_1$ and $\sigma^2{}_2$ are UnKnowns and $\sigma^2{}_1 \neq \sigma^2{}_2$

$$(\overline{x_1} - \overline{x_2}) - t_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{S^2{}_1}{n_1}\right) + \left(\frac{S^2{}_2}{n_2}\right)} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + t_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{S^2{}_1}{n_1}\right) + \left(\frac{S^2{}_2}{n_2}\right)}$$

$$\text{Degrre fredom} : \Upsilon = \frac{\left[\frac{S^2{}_1}{n_1} + \frac{S^2{}_2}{n_2}\right]^2}{\left[\left(\frac{S^2{}_1}{n_1}\right)/n_1 - 1\right]\left[\left(\frac{S^2{}_2}{n_2}\right)/n_2 - 1\right]}$$

x_1, x_2 : *sample averages* ; n_1, n_2 : *sample sizes* ; S_P: *pooled Estimate*

σ_1, σ_2 : *population standerd devration*

▪**Single Sample** : **Estimating a Proportion** :

$$\hat{P} - Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\frac{\hat{p}\hat{q}}{n}} < P < \hat{P} + Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\frac{\hat{p}\hat{q}}{n}} \;\; ; \quad \hat{P} = \frac{x}{n} = \frac{number\ of\ sucess}{Total\ sample\ size}$$

⇨ $\hat{q} = 1 - \hat{p}$

▪***Error*** (∈) : $Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

▪***Sample Size*** ($\boldsymbol{n}$) : $\left(\frac{Z_{\left(\frac{\alpha}{2}\right)}}{\epsilon}\right)^2 . \hat{p}\hat{q}$

▪**Two Samples** : **Estimating the difference between Proportions** :

$$\hat{P} - Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\frac{\hat{p}\hat{q}}{n}} < P < \hat{P} + Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\left(\widehat{P_1} - \widehat{P_2}\right) - Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\left(\frac{\widehat{p_1}\,\widehat{q_1}}{n_1}\right) + \left(\frac{\widehat{p_2}\,\widehat{q_2}}{n_2}\right)} < P_1 - P_2$$

$$< \left(\widehat{P_1} - \widehat{P_2}\right) + Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\left(\frac{\widehat{p_1}\,\widehat{q_1}}{n_1}\right) + \left(\frac{\widehat{p_2}\,\widehat{q_2}}{n_2}\right)}$$

Exercises

<u>9.4 Page 285</u> :

$Given: \ \sigma = 40 \ \ , \bar{x} = 780 \ , S = 5 \ , n = 30, \alpha = 96\% = 0.04$

$\left(\frac{\alpha}{2}\right) = \frac{0.04}{2} = 0.02 \ ; \ Z_{0.02} \ = 0.5080$

$\bar{x} - Z_{\left(\frac{\alpha}{2}\right)} \frac{\mu}{\sqrt{n}} < \mu \ < \ \bar{x} + Z_{\left(\frac{\alpha}{2}\right)} \frac{\mu}{\sqrt{n}} \quad we\ use\ this\ rule\ if\ n \geq 30$

$= \ 780 - 0.5080\left(\frac{40}{\sqrt{30}}\right) < \mu \ < \ \ 780 + 0.5080\left(\frac{40}{\sqrt{30}}\right) \ = 776.29 \ < \mu \ < 783.7$

<u>9.8 Page 286</u> :

$Given: \ \sigma = 40 \ \ , \bar{x} = 780 \ , S = 5 \ , n = 30, \alpha = 96\% = 0.04$

$\left(\frac{\alpha}{2}\right) = \frac{0.04}{2} = 0.02 \ ; \ Z_{0.02} \ = 2.054$

⇨ $776.29 \ < \mu \ < 783.7$

⇨ $Error\ (\in): \ \ Z_{\left(\frac{\alpha}{2}\right)} \frac{\mu}{\sqrt{n}} = 2.054 \ \left(\frac{40}{\sqrt{30}}\right) = 15$

⇨ $Sample\ Size\ (n): \ \ \left(\frac{Z_{\left(\frac{\alpha}{2}\right)} \ \sigma}{\in}\right)^2 = \left(\frac{2.054 \times 40}{15}\right)^2 \ = 30$

Find the sample size if we want error = 10 % ?

⇨ $Sample\ Size\ (n): \ \ \left(\frac{Z_{\left(\frac{\alpha}{2}\right)} \ \sigma}{\in}\right)^2 = \left(\frac{2.054 \times 40}{10}\right)^2 \ = 67.5$

<u>9.12 Page 286</u> :

$Given: \ \sigma = ? \ , \bar{x} = 11.3 \ , S = 2.45 \ , n = 20, \alpha = 95\% = 0.05 \ , t_{0.025} \ = 2.093$

$\bar{x} - t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}} < \mu \ < \ \bar{x} + t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}} \quad we\ use\ this\ rule\ if\ n < 30$

$= \ 11.3 - 2.093\left(\frac{2.45}{\sqrt{20}}\right) < \mu \ < \ 11.3 + 2.093\left(\frac{2.45}{\sqrt{20}}\right) \ = 10.15 \ < \mu \ < 12.45$

<u>9.13 Page 286</u> :

$Given: \ \ n = 9, \alpha = 99\% = 0.01, \left(\frac{\alpha}{2}\right) = \frac{0.01}{2} = 0.005 \ ; \ t_{0.005} \ = 3.355$

⇨ $\bar{x} = \frac{1.01 + 0.97 + 1.03 + 1.04 + 0.99 + 0.99 + 0.98 + 1.01 + 1.03}{9} = 1.005$

<table>
<tr><th>x</th><th>$x - \bar{x}$</th><th>$(x - \bar{x})^2$</th><th>$\bar{x} = 1.005$</th></tr>
<tr><td>1.01</td><td>0.005</td><td>0.000025</td><td rowspan="5">$\sum (x - \bar{x})^2 = 0.04551$
$S^2 = \frac{0.04551}{8} = 0.00568$</td></tr>
<tr><td>0.97</td><td>- 0.035</td><td>0.001255</td></tr>
<tr><td>1.03</td><td>-0.175</td><td>0.030</td></tr>
<tr><td>1.04</td><td>0.035</td><td>0.001255</td></tr>
<tr><td>0.99</td><td>- 0.015</td><td>0.000225</td></tr>
<tr><td>0.98</td><td>- 0.025</td><td>0.00625</td><td rowspan="3">$S = \sqrt{0.00568} = 0.075$</td></tr>
<tr><td>1.01</td><td>0.005</td><td>0.000025</td></tr>
<tr><td>1.03</td><td>0.025</td><td>0.00625</td></tr>
</table>

0.99	- 0.015	0.000225	

$\bar{x} - t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{\left(\frac{\alpha}{2}\right)} \frac{S}{\sqrt{n}}$ *we use this rule if* $n < 30$

$= 1.005 - 3.355 \left(\frac{0.075}{\sqrt{9}}\right) < \mu < 1.005 + 3.355 \left(\frac{0.075}{\sqrt{9}}\right) = 0.92 < \mu < 1.08$

9.35 Page 297 :

$Given: \overline{x_1} = 80, \overline{x_2} = 75, n_1 = 25, n_2 = 36, \sigma_1 = 5, \sigma_2 = 3, \alpha = 94\% = 0.06$

$\left(\frac{\alpha}{2}\right) = \frac{0.06}{2} = 0.03$; $Z_{0.03} = 1.88$

$$(\overline{x_1} - \overline{x_2}) - Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{\sigma^2{}_1}{n_1}\right) + \left(\frac{\sigma^2{}_2}{n_2}\right)} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{\sigma^2{}_1}{n_1}\right) + \left(\frac{\sigma^2{}_2}{n_2}\right)}$$

$$(80 - 75) - 1.88 \sqrt{\left(\frac{25}{25}\right) + \left(\frac{9}{36}\right)} < \mu_1 - \mu_2 < (80 - 75) + 1.88 \sqrt{\left(\frac{25}{25}\right) + \left(\frac{9}{36}\right)}$$

$= 2.89 < \mu_1 - \mu_2 < 7.10$

9.38 Page 297 :

$Given: \overline{x_1} = 85, \overline{x_2} = 81, n_1 = 12, n_2 = 10, S_1 = 4, S_2 = 5, \alpha = 90\% = 0.10$

$\left(\frac{\alpha}{2}\right) = \frac{0.10}{2} = 0.05$; $t_{0.05} = 1.723$

$$S_P = \sqrt{\frac{(n_1 - 1)S^2{}_1 + (n_2 - 1)S^2{}_2}{n_1 + n_2 - 2}} = S_P = \sqrt{\frac{(12 - 1)(16) + (10 - 1)(25)}{12 + 10 - 2}} = 4.477$$

$$(\overline{x_1} - \overline{x_2}) - t_{\left(\frac{\alpha}{2}\right)} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + t_{\left(\frac{\alpha}{2}\right)} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(85 - 81) - 1.723(4.477) \sqrt{\left(\frac{1}{12}\right) + \left(\frac{1}{10}\right)} < \mu_1 - \mu_2 < (85 - 81) + 1.723(4.477) \sqrt{\left(\frac{1}{12}\right) + \left(\frac{1}{10}\right)}$$

$= 0.697 < \mu_1 - \mu_2 < 7.30$

9.46 Page 298 :

⇨ $\overline{x_1} = \frac{103 + 94 + 110 + 87 + 98}{5} = 98.4$

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\overline{x_1} = 98.4$
87	-11.4	129.96	$\sum (x - \bar{x})^2 = 305.2$ $S^2 = \frac{305.2}{4} = 76.3$
94	-4.4	19.36	
98	-0.4	0.16	
103	4.6	21.16	
110	11.6	134.56	$S = \sqrt{76.3} = 8.735$

⇨ $\overline{x_2} = \frac{97 + 82 + 123 + 92 + 175 + 88 + 118}{7} = 110.71$

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\overline{x_2} = 110.71$
82	-28.71	824.26	$\sum (x - \bar{x})^2 = 6215.30$
88	-22.71	515.74	
92	-18.71	350	

97	-13.71	187.96	$S^2 = \frac{6215.30}{6} = 1035.88$
118	7.29	53.14	
123	12.29	151	$S = \sqrt{1035.88} = 32.185$
175	64.29	4133.2	

$$(\overline{x_1} - \overline{x_2}) - t_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{S^2{}_1}{n_1}\right) + \left(\frac{S^2{}_2}{n_2}\right)} < \mu_1 - \mu_2 < (\overline{x_1} - \overline{x_2}) + t_{\left(\frac{\alpha}{2}\right)} \sqrt{\left(\frac{S^2{}_1}{n_1}\right) + \left(\frac{S^2{}_2}{n_2}\right)}$$

$$(98.4 - 110.7) - 1.892 \sqrt{\left(\frac{8.735^2}{5}\right) + \left(\frac{32.185^2}{7}\right)} < \mu_1 - \mu_2$$

$$< (98.4 - 110.7) + 1.892 \sqrt{\left(\frac{8.735^2}{5}\right) + \left(\frac{32.185^2}{7}\right)} = -36.5 < \mu_1 - \mu_2 < 11.87$$

9.51 Page 304 :

A) $\hat{P} = \frac{x}{n} = \frac{114}{200} = 0.57$ ⇨ $\hat{q} = 1 - \hat{p} = 1 - 0.57 = 0.43$ ⇨ $Z_{0.02} = 2.054$

$$\hat{P} - Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\frac{\hat{p}\hat{q}}{n}} < P < \hat{P} + Z_{\left(\frac{\alpha}{2}\right)} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.57 - 2.054 \sqrt{\frac{(0.57)(0.43)}{200}} < P < 0.57 + 2.054 \sqrt{\frac{(0.57)(0.43)}{200}} = 0.49 < P < 0.64$$

B) $Error\ (\in) = 2.054\sqrt{\frac{(0.57)(0.43)}{200}} = 0.072$

9.52 Page 304 :

$\hat{P} = \frac{x}{n} = \frac{485}{500} = 0.97$ ⇨ $\hat{q} = 1 - \hat{p} = 1 - 0.97 = 0.03$ ⇨ $Z_{0.05} = 1.645$

$$0.97 - 1.645 \sqrt{\frac{(0.97)(0.03)}{500}} < P < 0.97 + 1.645 \sqrt{\frac{(0.97)(0.03)}{500}} = 0.957 < P < 0.983$$

9.53 Page 304 :

$\hat{P} = \frac{x}{n} = \frac{228}{1000} = 0.228$ ⇨ $\hat{q} = 1 - \hat{p} = 1 - 0.228 = 0.772$ ⇨ $Z_{0.005} = 2.575$

$$0.228 - 2.575 \sqrt{\frac{(0.228)(0.772)}{1000}} < P < 0.228 + 2.575 \sqrt{\frac{(0.228)(0.772)}{1000}}$$

$= 0.194 < P < 0.262$

9.59 Page 305 :

$Error\ (\in) = 0.02 = 2\ \%$; $Sample\ Size\ (n) : \left(\frac{Z_{\left(\frac{\alpha}{2}\right)}}{\in}\right)^2 . \hat{p}\hat{q}$

$$n = \left(\frac{2.054}{0.02}\right)^2 \times (0.57)(0.43) = 2575$$

9.60 Page 305 :

$$Error\ (\in) = \quad 0.02 = 2\ \% \ \ n = \left(\frac{2.575}{0.05}\right)^2 \times (0.228)(0.772) = 467$$

9.65 Page 305 :

$$\hat{P}_1 = \frac{x_1}{n_1} = \frac{250}{1000} = 0.25 \quad \Rightarrow \hat{q}_1 = 1 - \hat{P}_1 = 1 - 0.25 = 0.75 \quad \Rightarrow Z_{0.025} = 1.96$$

$$\hat{P}_2 = \frac{x_2}{n_2} = \frac{275}{1000} = 0.275 \quad \Rightarrow \hat{q}_2 = 1 - \hat{P}_2 = 1 - 0.275 = 0.725$$

$$\left(\widehat{P_1} - \widehat{P_2}\right) - Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\left(\frac{\widehat{p_1}\,\widehat{q_1}}{n_1}\right) + \left(\frac{\widehat{p_2}\,\widehat{q_2}}{n_2}\right)} < P_1 - P_2 < \left(\widehat{P_1} - \widehat{P_2}\right) + Z_{\left(\frac{\alpha}{2}\right)}\sqrt{\left(\frac{\widehat{p_1}\,\widehat{q_1}}{n_1}\right) + \left(\frac{\widehat{p_2}\,\widehat{q_2}}{n_2}\right)}$$

$$(0.25 - 0.275) - 1.96\sqrt{\left(\frac{(0.25)(0.75)}{1000}\right) + \left(\frac{(0.275)(0.725)}{1000}\right)} < P_1 - P_2$$

$$< (0.25 - 0.275) + 1.96\sqrt{\left(\frac{(0.25)(0.75)}{1000}\right) + \left(\frac{(0.275)(0.725)}{1000}\right)}$$

$$= -0.0635 < P_1 - P_2 < 0.0135$$

9.66 Page 305 :

$$\hat{P}_1 = \frac{x_1}{n_1} = \frac{80}{250} = 0.32 \quad \Rightarrow \hat{q}_1 = 1 - \hat{P}_1 = 1 - 0.32 = 0.68 \quad \Rightarrow Z_{0.05} = 1.645$$

$$\hat{P}_2 = \frac{x_2}{n_2} = \frac{40}{175} = 0.2286 \quad \Rightarrow \hat{q}_2 = 1 - \hat{P}_2 = 1 - 0.2286 = 0.7714$$

$$(0.32 - 0.2286) - 1.645\sqrt{\left(\frac{(0.32)(0.68)}{250}\right) + \left(\frac{(0.2286)(0.7714)}{175}\right)} < P_1 - P_2$$

$$< (0.32 - 0.2286) + 1.645\sqrt{\left(\frac{(0.32)(0.68)}{250}\right) + \left(\frac{(0.2286)(0.7714)}{175}\right)}$$

$$= 0.0011 < P_1 - P_2 < 0.0869$$

Chapter 10

▪**Test of Hypothesis** :

1)

H_O : Null Hypothesis	H_A : Alternate Hypothesis
One Tail	Two Tail
$H_O : \mu \leq 30$	$H_O : \mu = 30$
$H_A : \mu > 30$	$H_A : \mu \neq 30$

2) Test statistics (single mean) :

σ is Known	σ is Unknown
$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$	$t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)}$

3) Critical Values of z or t

4) Comparisons

One Tail	Two Tail
$t.s > c.v$ Reject H_O Test statics > Critcal values	$t.s\ z > Z_{\frac{\alpha}{2}}$; $t.s\ z < Z_{\frac{\alpha}{2}}$ Reject H_O

5) Conclusions

▪**Example10.1 Page 333**:

$H_O : \mu \leq 15$; $H_A : \mu > 15$

▪**Example10.2 Page 333**:

$H_O : \mu = 0.6$; $H_A : \mu \neq 0.6$

▪**Example10.3 Page 340**:

1) $H_O : \mu \leq 70$; $H_A : \mu > 70$ (*one tail*)
2) *Test statictcs* : $Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{71.8-70}{\frac{8.9}{\sqrt{100}}} = 2.022$
3) *Crictial values* : $\alpha = 0.05 \therefore Z_{0.05} = 1.65$
4) *Comparision* : $t.s > c.v = 2.022 > 1.65 \therefore$ *Reject* H_O
5) *Conclusions* : *Reject* H_O , *the average live time is grater than* 70

▪**Example10.4 Page 340**:

1) $H_O : \mu = 8$; $H_A : \mu \neq 8$ (*Two tail*)
2) *Test statictcs* : $Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{7.8-8}{\frac{0.5}{\sqrt{50}}} = -2.83$
3) *Crictial values* : $\alpha = 0.01$; $\frac{\alpha}{2} = 0.005 \therefore Z_{0.005} = 2.573$
4) *Comparision* : $t.s\ z > Z_{\frac{\alpha}{2}} = -2.83 > 2.575$ (*wrong*)

   $t.s\ z < Z_{\frac{\alpha}{2}} = -2.83 < -2.575$ (*correct*) $\therefore$ *Reject* H_O
5) *Conclusions* : *Reject* H_O , *the average live time is not equal* 8

Exercises

10.25 Page 357 :

⇨ $\overline{x_2} = \dfrac{9.7 + 9.8 + 9.8 + 9.9 + 10.1 + 10.1 + 10.2 + 10.3 + 10.3 + 10.4}{10} = 10$

x	$x - \bar{x}$	$(x - \bar{x})^2$	$\overline{x_2} = 10$
9.7	-0.3	0.09	$\sum(x - \bar{x})^2 = 3.76$ $S^2 = \frac{3.76}{9} = 0.417$
9.8	-0.2	0.04	
9.8	-0.2	0.04	
9.9	-0.1	0.01	
10.1	0.1	0.01	
10.1	0.1	0.01	
10.2	0.2	0.04	$S = \sqrt{0.417} = 0.65$
10.3	0.3	0.09	
10.3	0.3	0.09	
10.4	0.4	0.016	

1) $H_O : \mu = 10 \quad ; \quad H_A : \mu \neq 10 \quad (Two\ tail)$
2) $Test\ statictcs : Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{10.06-10}{\frac{0.246}{\sqrt{10}}} = 0.65$
3) $Crictial\ values : \alpha = 0.01 \ ; \ \frac{\alpha}{2} = 0.005 \ \therefore \ Z_{0.005} = 3.25$
4) $Comparision : \ t.s\ z > Z_{\frac{\alpha}{2}} = 0.65 > 3.25 \ (wrong)$
   $t.s\ z < Z_{\frac{\alpha}{2}} = 0.65 < -3.25 \ (wrong) \therefore Dont\ Reject\ H_O$
5) $Conclusions : Fail\ toReject\ H_O$

10.26 Page 357 :

1) $H_O : \mu \leq 220 \quad ; \quad H_A : \mu > 220 \ (one\ tail)$
2) $Test\ statictcs : Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{244-220}{\frac{24.5}{\sqrt{20}}} = 4.38$
3) $Crictial\ values : \alpha = 0.01 \ \therefore \ Z_{0.01} = 1.729$
4) $Comparision : \ t.s > c.v = 4.38 > 1.729 \ \therefore Reject\ H_O$
5) $Conclusions : Reject\ H_O, and\ claim\ \mu > 220$

Chapter 11

▪**Linear Regression Line** :

⇨ $\hat{y} = b_0 + b_1 x$; $b_0: y - intersept$; $b_1: Slope\ of\ the\ line$		
$\bar{x} = \frac{\sum xi}{n}$	$\bar{y} = \frac{\sum yi}{n}$	$b_0 = \bar{y} - b_1 \bar{x}$
$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$	$b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$	

▪**Correlation Coefficient** :

$\gamma = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$	$\gamma = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum(x)^2\ (\sum x)^2]\,[n\sum(y)^2\ (\sum y)^2]}}$

▪**Example①**: For the following 12 data find the linear regression line ?

x	y	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$
40	385	5.833	34	-73.75	-430.18
20	400	-14.16	200.5	-58.75	831.9
25	395	-9.167	84	-63.75	584.4
20	365	-14.16	200.5	-93.75	562.86
30	475	-4.167	17.36	16.25	-67.71
50	440	15.833	250.7	-18.75	-296.9
40	490	5.833	34	31.25	182.3
20	420	-14.16	200.5	-38.75	548.7
50	560	15.833	250.7	101.25	1603
40	525	5.833	34	66.25	386.44
25	485	-9.167	84	26.25	-240.63
50	570	15.833	250.7	111.25	1761.42
410	5505		1641		5425.6

$\bar{x} = \frac{\sum xi}{n} = \frac{410}{12} = 34.167$; $\bar{y} = \frac{\sum yi}{n} = \frac{5505}{12} = 458.75$

⇨ $b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{5425.6}{1641} = 3.30$

⇨ $b_0 = \bar{y} - b_1 \bar{x} = 458.75 - (3.30)(34.167) = 346$

⇨ $\hat{y} = b_0 + b_1 \bar{x} = 458 + 346\,x$

▪**Example②**: For the 12 data Find from following information linear regression line $\sum xi = 311.6$; $\sum yi = 297.2$; $\sum xi^2 = 8134.26$; $\sum xy = 7568.76$?

$\bar{x} = \frac{\sum xi}{n} = \frac{311.6}{12} = 25.96$; $\bar{y} = \frac{\sum yi}{n} = \frac{297.2}{12} = 24.76$

$$\Rightarrow b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{7568.76 - \frac{(311.6)(297.2)}{12}}{8134.26 - \frac{(311.6)^2}{12}} = -3.45$$

$$\Rightarrow b_0 = \bar{y} - b_1 \bar{x} = 24.76 - (-3.45)(25.96) = -114.33$$

$$\Rightarrow \hat{y} = b_0 + b_1 \bar{x} = 114.33 - 3.45\, x$$

▪**Example①**: For the following 5 data find the correlation coefficient ?

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1	12	-2	-19.8	4	392	39.6
2	24	-1	-7.8	1	60.84	7.8
3	32	0	0.2	0	0.04	0
4	41	1	9.2	1	84.64	9.2
5	50	2	18.2	4	331.24	36.4
15	159			10	868.8	93

$$\bar{x} = \frac{\sum xi}{n} = \frac{15}{5} = 3 \ ; \ \bar{y} = \frac{\sum yi}{n} = \frac{159}{5} = 31.8$$

$$\Rightarrow \gamma = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}} = \frac{93}{\sqrt{(10)(868.8)}} = 0.99$$

▪**Example②**: For the 9 data Find from following information correlation coefficient $\sum xi = 20$; $\sum yi = 30$; $\sum xi^2 = 1400$; $\sum yi^2 = 3600$; $\sum xy = 500$?

$$\bar{x} = \frac{\sum xi}{n} = \frac{20}{9} = 2.22 \ ; \ \bar{y} = \frac{\sum yi}{n} = \frac{30}{9} = 30.33$$

$$\Rightarrow \gamma = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum(x)^2 \ (\sum x)^2]\,[n\sum(y)^2 \ (\sum y)^2]}}$$

$$= \frac{(9)(500) - (20)(30)}{\sqrt{[9(1400) - 20^2]\,[9(3600) - 30^2\,]}} = 0.26$$