

## حل تمارين المحاظره الثاني عشر

أوجد ناتج التكاملات الآتية:

$$i. \int (5x^6 - 2x^4 + 3x^2 - 6) dx$$

: الحل

$$\begin{aligned}\int (5x^6 - 2x^4 + 3x^2 - 6) dx &= \frac{5x^7}{7} - \frac{2x^5}{5} + \frac{3x^3}{3} - 6x + c \\ &= \frac{5x^7}{7} - \frac{2x^5}{5} + x^3 - 6x + c\end{aligned}$$



$$ii. \int (x^{1/2} - 3x^{2/3} + 5x^{-1/2}) dx$$

: الحل

$$\begin{aligned}\int \left( x^{\frac{1}{2}} - 3x^{\frac{2}{3}} + 5x^{-\frac{1}{2}} \right) dx &= \frac{x^{\frac{3}{2}}}{3/2} - 3 \frac{x^{\frac{5}{3}}}{5/3} + 5 \frac{x^{\frac{1}{2}}}{1/2} + c \\ &= \frac{2}{3}x^{\frac{3}{2}} - \frac{9}{5}x^{\frac{5}{3}} + 10x^{\frac{1}{2}} + c\end{aligned}$$





$$iii. \int 2x dx$$

الحل:

$$\begin{aligned}\int 2x dx &= \frac{2x^2}{2} + c \\ &= x^2 + c\end{aligned}$$



$$v. \int (3 \cos x + 2x) dx$$

الحل:

$$\begin{aligned}\int (3 \cos x + 2x) dx &= 3 \sin x + \frac{2x^2}{2} + c \\ &= 3 \sin x + x^2 + c\end{aligned}$$



$$vii. \quad \int -2e^x dx$$

الحل:

$$\int -2e^x dx = -2e^x + c$$



$$viii. \quad \int \frac{x^5 + 2}{x^3} dx$$

الحل:

$$\begin{aligned} \int \frac{x^5 + 2}{x^3} dx &= \int (x^2 + 2x^{-3}) dx = \frac{x^3}{3} + 2 \frac{x^{-2}}{(-2)} + c \\ &= \frac{x^3}{3} - x^{-2} + c \end{aligned}$$



## حل تمارين المحاظره الثالث عشر

أوجد التكاملات التالية:  
الحل:

$$\begin{aligned} u &= 3x \\ du &= 3dx \end{aligned}$$

$$\begin{aligned} \int \cos 3x \, dx &= \frac{1}{3} \int 3 \cos 3x \, du = \\ &= \frac{1}{3} \int \cos u \, du = \frac{1}{3} \sin u + c = \frac{1}{3} \sin 3x + c \end{aligned}$$

الحل:

$$\begin{aligned} u &= 2x \\ du &= 2dx \end{aligned}$$

$$\begin{aligned} \int e^{2x} \, dx &= \frac{1}{2} \int 2e^{2x} \, du = \\ &= \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + c = \frac{1}{2} e^{2x} + c \end{aligned}$$

الحل:

$$\int \frac{x^2 dx}{x^3 + 1} , x \neq -1$$

$$\int \frac{x^2 dx}{x^3 + 1} = \int x^2 (x^3 + 1)^{-1} dx$$

$$\begin{aligned} u &= x^3 + 1 \\ du &= 3x^2 dx \end{aligned}$$

$$\int x^2 (x^3 + 1)^{-1} dx = \frac{1}{3} \int 3x^2 (x^3 + 1)^{-1} du = \frac{1}{3} \int u^{-1} du = \frac{1}{3} \ln|u| + c = \frac{1}{3} \ln(x^3 + 1) + c$$

حل المعادلة التفاضلية المعطاة:

$$\frac{dy}{dx} = 2x + 3$$

الحل:

$$dy = (2x + 3) dx$$

$$\begin{aligned} \int dy &= \int (2x + 3) dx \\ y &= \frac{2x^2}{2} + 3x + c \\ \therefore y &= x^2 + 3x + c \end{aligned}$$

## حل المعادلة التفاضلية المعطاة:

$$\frac{dy}{dx} = \frac{x}{y}$$

الحل:

$$y \ dy = x \ dx$$

$$\int y \ dy = \int x \ dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c$$

$$\frac{dy}{dx} = \sqrt{xy}$$

الحل:

$$\frac{dy}{dx} = x^{\frac{1}{2}} y^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$\int y^{-\frac{1}{2}} dy = \int x^{\frac{1}{2}} dx$$

$$\frac{y^{\frac{1}{2}}}{1/2} = \frac{x^{\frac{3}{2}}}{3/2} + c$$

$$2 y^{\frac{1}{2}} = \frac{2}{3} x^{\frac{3}{2}} + c$$

## حل تمارين المحاظره الرابعة عشر

أوجد التكاملات التالية:

$$i. \int_0^2 (5x^3 - 3x + 6)dx$$

الحل:

$$\begin{aligned}\int_0^2 (5x^3 - 3x + 6)dx &= \left[ \frac{5x^4}{4} - \frac{3x^2}{2} + 6x \right]_0^2 \\ &= \left[ \frac{5(2)^4}{4} - \frac{3(2)^2}{2} + 6(2) \right] - 0 \\ &= [20 - 6 + 12] = 26\end{aligned}$$



$$ii. \int_{-2}^3 7dx$$

الحل:

$$\begin{aligned}\int_{-2}^3 7dx &= [7x]_{-2}^3 \\ &= [7(3)] - [7(-2)] \\ &= 21 - (-14) = 21 + 14 = 35\end{aligned}$$





$$iii. \int_{4}^{4} (x - 16) dx$$

الحل:

$$\int_{4}^{4} (x - 16) dx = 0$$



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$$vi. \int_{0}^{\pi} \cos x dx$$

الحل:

$$\int_{0}^{\pi} \cos x dx = [\sin x]_0^{\pi} = (\sin \pi) - (\sin 0) = 0$$



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الحل:

$$ix. \int_0^{\pi} \sec^2 x \, dx$$

$$\begin{aligned}\int_0^{\pi} \sec^2 x \, dx &= [\tan x]_0^{\pi} \\ &= \tan \pi - \tan 0 = 0\end{aligned}$$



الحل:

$$v. \int_{-1}^2 (x^3 + 1)^2 \, dx$$

$$\begin{aligned}\int_{-1}^2 (x^3 + 1)^2 \, dx &= \int_{-1}^2 (x^6 + 2x^3 + 1) \, dx \\ &= \left[ \frac{x^7}{7} + \frac{2x^4}{4} + x \right]_{-1}^2 \\ &= \left[ \frac{(2)^7}{7} + \frac{2(2)^4}{4} + 2 \right] - \left[ \frac{(-1)^7}{7} + \frac{2(-1)^4}{4} + (-1) \right]\end{aligned}$$





$$\begin{aligned}&= \left[ \frac{128}{7} + 8 + 2 \right] - \left[ \frac{-1}{7} + \frac{1}{2} - 1 \right] \\&= \frac{128}{7} + \frac{1}{7} - \frac{1}{2} + 11 \\&= \frac{129}{7} - \frac{1}{2} + 11 \\&= \frac{258}{14} - \frac{7}{14} + \frac{154}{14} = \frac{405}{14}\end{aligned}$$

