حل تمارين المحاظره الثاني عشر

أوجد ناتج التكاملات الآتية:

$$i.\ \int\left(5x^{6}-2x^{4}+3x^{2}-6\right)dx$$

الحل:

$$\int\left(5x^{6}-2x^{4}+3x^{2}-6\right)dx=\frac{5x^{7}}{7}-\frac{2x^{5}}{5}+\frac{3x^{3}}{3}-6x+c$$

$$=\frac{5x^{7}}{7}-\frac{2x^{5}}{5}+x^{3}-6x+c$$

$$ii.\ \int\left(x^{1/2}-3x^{2/3}+5x^{-1/2}\right)dx$$

الحل:

$$\int\left(x^{\frac{1}{2}}-3x^{\frac{2}{3}}+5x^{-\frac{1}{2}}\right)dx=\frac{x^{\frac{3}{2}}}{3/2}-3\frac{x^{\frac{5}{3}}}{5/3}+5\frac{x^{\frac{1}{2}}}{1/2}+c$$

$$=\frac{2}{3}x^{\frac{3}{2}}-\frac{9}{5}x^{\frac{5}{3}}+10x^{\frac{1}{2}}+c$$

$$iii. \quad \int 2x dx$$

الحل:

$$\int 2x\, dx = \frac{2x^2}{2} + c$$
$$= x^2 + c$$

$$v. \quad \int (3\cos x + 2x) dx$$

الحل:

$$\int (3\cos x + 2x) dx = 3\sin x + \frac{2x^2}{2} + c$$
$$= 3\sin x + x^2 + c$$

$$vii. \quad \int -2e^x dx$$

الحل:

$$\int -2e^x dx = -2e^x + c$$

$$viii. \quad \int \frac{x^5+2}{x^3} dx$$

الحل:

$$\int \frac{x^5+2}{x^3} dx = \int \left(x^2 + 2x^{-3}\right) dx = \frac{x^3}{3} + 2\frac{x^{-2}}{(-2)} + c$$

$$= \frac{x^3}{3} - x^{-2} + c$$

حل تمارين المحاظره الثالث عشر

أوجد التكاملات التالية:

$$\int \cos 3x \, dx$$

الحل:

$$u = 3x$$
$$du = 3dx$$

$$\int \cos 3x \, dx = \frac{1}{3}\int 3\cos 3x \, du =$$

$$= \frac{1}{3}\int \cos u \, du \quad = \frac{1}{3}\sin u + c = \frac{1}{3}\sin 3x + c$$

$$\int e^{2x} dx$$

الحل:

$$u = 2x$$
$$du = 2\,dx$$

$$\int e^{2x} \, dx = \frac{1}{2}\int 2e^{2x} \, du =$$

$$= \frac{1}{2}\int e^{u} \, du \quad = \frac{1}{2}e^{u} + c = \frac{1}{2}e^{2x} + c$$

الحل:

$$\int \frac{x^2 dx}{x^3+1} \quad , x \neq -1$$

$$\int \frac{x^2 dx}{x^3+1} = \int x^2 (x^3+1)^{-1} dx$$

$$u = x^3 + 1$$
$$du = 3x^2 \, dx$$

$$\int x^2(x^3+1)^{-1} dx = \frac{1}{3}\int 3x^2(x^3+1)^{-1} du = \frac{1}{3}\int u^{-1} du \quad = \frac{1}{3}\ln|u| + c = \frac{1}{3}\ln\left(x^3+1\right) + c$$

حل المعادلة التفاضلية المعطاة:

$$\frac{dy}{dx} = 2x + 3$$

الحل:

$$dy = (2x+3)\,dx$$

$$\int dy = \int (2x+3)\,dx$$

$$y = \frac{2x^2}{2} + 3x + c$$

$$\therefore \quad y = x^2 + 3x + c$$

حل المعادلة التفاضلية المعطاة:

$$\frac{dy}{dx} = \frac{x}{y}$$

الحل:

$$y\,dy = x\,dx$$

$$\int y \; dy = \int x \; dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c$$

$$\frac{dy}{dx} = \sqrt{xy}$$

الحل:

$$\frac{dy}{dx} = x^{\frac{1}{2}} y^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$\int y^{-\frac{1}{2}} dy = \int x^{\frac{1}{2}} dx$$

$$\frac{y^{\frac{1}{2}}}{1/2} = \frac{x^{\frac{3}{2}}}{3/2} + c$$

$$2\,y^{\frac{1}{2}} = \frac{2}{3} x^{\frac{3}{2}} + c$$

حل تمارين المحاظره الرابعة عشر

أوجد التكاملات التالية :

$$i. \quad \int_0^2 (5x^3 - 3x + 6)dx$$

الحل:

$$\int_0^2 (5x^3 - 3x + 6)dx = \left[\frac{5x^4}{4} - \frac{3x^2}{2} + 6x\right]_0^2$$

$$= \left[\frac{5(2)^4}{4} - \frac{3(2)^2}{2} + 6(2)\right] - 0$$

$$= [20 - 6 + 12] = 26$$

$$ii. \quad \int_{-2}^{3} 7dx$$

الحل:

$$\int_{-2}^{3} 7dx = [7x]_{-2}^{3}$$

$$= [7(3)] - [7(-2)]$$

$$= 21 - (-14) = 21 + 14 = 35$$

$$iii. \quad \int_{4}^{4} (x-16)\, dx$$

الحل:

$$\int_{4}^{4} (x-16)\, dx = 0$$

$$vi. \quad \int_{0}^{\pi} \cos x\, dx$$

الحل:

$$\int_{0}^{\pi} \cos x\, dx = [\sin x]_{0}^{\pi} = (\sin \pi) - (\sin 0) = 0$$

$$ix.\quad \int_0^{\pi} \sec^2 x\, dx$$

الحل:

$$\int_0^{\pi} \sec^2 x\, dx = [\tan x]_0^{\pi}$$

$$= \tan \pi - \tan 0 = 0$$

$$v.\quad \int_{-1}^{2} (x^3+1)^2 dx$$

الحل:

$$\int_{-1}^{2} (x^3+1)^2 dx = \int_{-1}^{2} (x^6 + 2x^3 + 1) dx$$

$$= \left[\frac{x^7}{7} + \frac{2x^4}{4} + x \right]_{-1}^{2}$$

$$= \left[\frac{(2)^7}{7} + \frac{2(2)^4}{4} + 2 \right] - \left[\frac{(-1)^7}{7} + \frac{2(-1)^4}{4} + (-1) \right]$$

$$= \left[\frac{128}{7} + 8 + 2\right] - \left[\frac{-1}{7} + \frac{1}{2} - 1\right]$$

$$= \frac{128}{7} + \frac{1}{7} - \frac{1}{2} + 11$$

$$= \frac{129}{7} - \frac{1}{2} + 11$$

$$= \frac{258}{14} - \frac{7}{14} + \frac{154}{14} = \frac{405}{14}$$

تمت بحمد الله