

تمارين المحاضرة الثالثة.

1. إذا كان $f(x) = x^2 + 5x - 2$ ، $g(x) = 3x - 2$ فأوجد:

$$(i) (f + g)(x) = (x^2 + 5x - 2) + (3x - 2) = x^2 + 8x - 4$$

$$(ii) (f - g)(x) = (x^2 + 5x - 2) - (3x - 2) \\ = x^2 + 5x - 2 - 3x + 2 = x^2 + 2x$$

$$(iii) (f \times g)(x) = (x^2 + 5x - 2)(3x - 2) \\ = 3x^3 - 2x^2 + 15x^2 - 10x - 6x + 4 \\ = 3x^3 + 13x^2 - 16x + 4$$

$$(iv) \frac{f}{g}(x) = \frac{x^2 + 5x - 2}{3x - 2}$$

2. إذا كان $f(x) = x^2 - 7x + 2$ ، $g(x) = x + 4$ فأوجد:

$$(i) (f + g)(x) = (x^2 - 7x + 2) + (x + 4) = x^2 - 6x + 6$$

$$(ii) (f - g)(x) = (x^2 - 7x + 2) - (x + 4) \\ = x^2 - 7x + 2 - x - 4 \\ = x^2 - 8x - 2$$

$$(iii) (f \cdot g)(x) = (x^2 - 7x + 2)(x + 4) \\ = x^3 + 4x^2 - 7x^2 - 28x + 2x + 8 \\ = x^3 - 3x^2 - 26x + 8$$

$$(iv) \frac{f}{g}(x) = \frac{x^2 - 7x + 2}{x + 4}$$

$$(v) (f \circ g)(x) = f(g(x)) = f(x + 4) \\ = (x + 4)^2 - 7(x + 4) + 2 \\ = x^2 + 8x + 16 - 7x - 28 + 2 \\ = x^2 + x - 10$$

$$(vi) (g \circ f)(x) = g(f(x)) = g(x^2 - 7x + 2) \\ = (x^2 - 7x + 2) + 4 \\ = x^2 - 7x + 6$$

3. إذا كانت $f(x) = 2x - 5$ ، $g(x) = 4x$ فأوجد:

$$\begin{aligned}(i) (f \circ g)(x) &= f(g(x)) = f(4x) \\ &= 2(4x) - 5 \\ &= 8x - 5\end{aligned}$$

$$\begin{aligned}(ii) (g \circ f)(x) &= g(f(x)) = g(2x - 5) \\ &= 4(2x - 5) \\ &= 8x - 20\end{aligned}$$

$$\begin{aligned}(iii) (f \circ g)(2) &= f(g(2)) = f(8) \\ &= 2(8) - 5 \\ &= 16 - 5 = 11\end{aligned}$$

$$\begin{aligned}(iv) (g \circ f)(5) &= g(f(5)) = g(2(5) - 5) = g(5) \\ &= 4(5) = 20\end{aligned}$$

4. أوجد معكوس الدوال الآتية:

$$(i) f(x) = -2x + 1$$

$$y = -2x + 1$$

$$x = -2y + 1$$

$$x - 1 = -2y \Rightarrow y = \frac{1 - x}{2}$$

$$\therefore f^{-1}(x) = \frac{1 - x}{2}$$

$$(ii) g(x) = 5x$$

$$y = 5x$$

$$x = 5y \Rightarrow y = \frac{x}{5}$$

$$\therefore g^{-1}(x) = \frac{x}{5}$$

$$(iii) f(x) = \frac{x-4}{3}$$

$$y = \frac{x-4}{3}$$

$$x = \frac{y-4}{3} \Rightarrow y-4 = 3x$$

$$y = 3x + 4$$

$$\therefore f^{-1}(x) = 3x + 4$$