١) ارسم المنطقة المحددة بالمنحنيات المعطاة أمام كل فقرة مما يأتي ثم أوجد مساحة المنطقة :

① $y = 4 - \frac{x^2}{3}$, $y = 0$, $x = 0$, $x = 3$

$y = 4 - \frac{x^2}{3} \Longrightarrow 3y = 12 - x^2 \Longrightarrow x^2 = 12 - 3y \Longrightarrow x^2 = 3(4-y)$

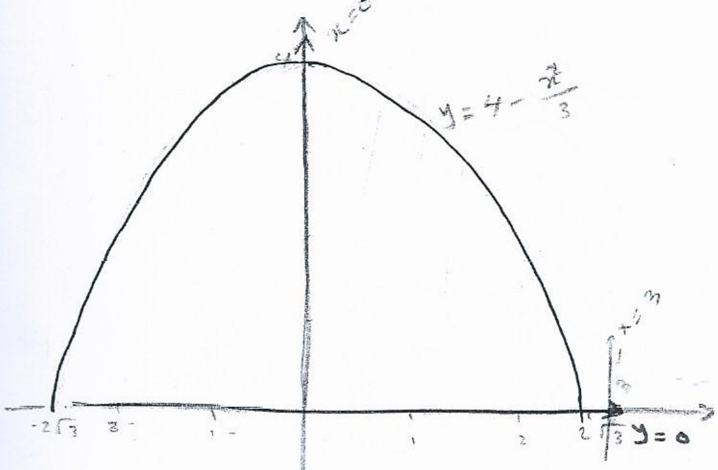
$x^2 = -3(y-4)$

تمثل معادلة قطع مكافئ متجه للأسفل رأسه $(0, 4)$

لإيجاد تقاطعها مع المحور السيني $y = 0$

$12 - x^2 = 0 \Longrightarrow x = \pm\sqrt{12} \Longrightarrow x = \pm 2\sqrt{3}$

$(2\sqrt{3}, 0)$ $(-2\sqrt{3}, 0)$



$$\int_0^3 \left(4 - \frac{x^2}{3}\right) dx = \left[4x - \frac{x^3}{9}\right]_0^3$$

$$= 12 - \frac{27}{9} = \frac{108 - 27}{9} = \frac{81}{9} = 9$$

② $y = x^2 + 3$, $y = x$, $x = -1$, $x = 1$

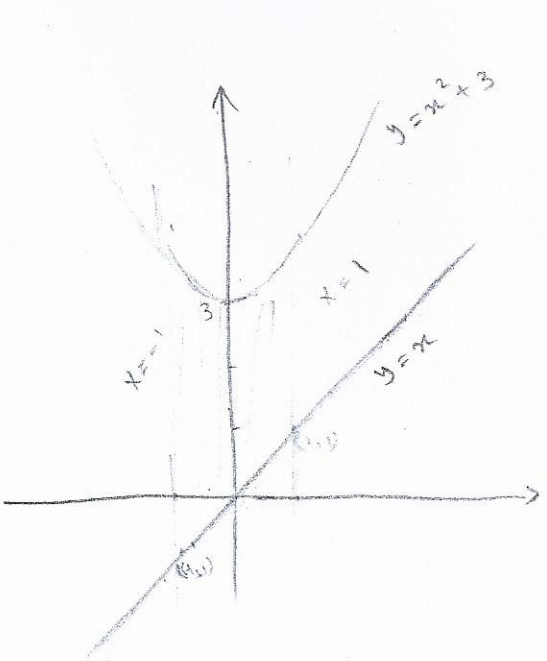
$y = x^2 + 3 \Longrightarrow x^2 = y - 3$

تمثل معادلة قطع مكافئ متجه للأعلى رأسه $(0, 3)$

لإيجاد التقاطع

$y = x \begin{cases} x = 1 \Longrightarrow y = 1 & (1, 1) \\ x = -1 \Longrightarrow y = & (-1, -1) \end{cases}$

$$y = x^2 + 3 \quad \begin{cases} x = 1 \implies y = 4 & (1,4) \\ x = -1 \implies y = 4 & (-1,4) \end{cases}$$



$$\int_{-1}^{1} (x^2 + 3 - x)\, dx$$

$$= \frac{x^3}{3} + 3x - \frac{x^2}{2} \Bigg]_{-1}^{1}$$

$$= \frac{1}{3} + 3 - \cancel{\frac{1}{2}} - \left(-\frac{1}{3} - 3 - \frac{1}{2}\right)$$

$$= \frac{2 + 18 - 3}{6} + \frac{2 + 18 + 3}{6}$$

$$= \frac{17 + 23}{6} = \frac{40}{6} = \frac{20}{3}$$

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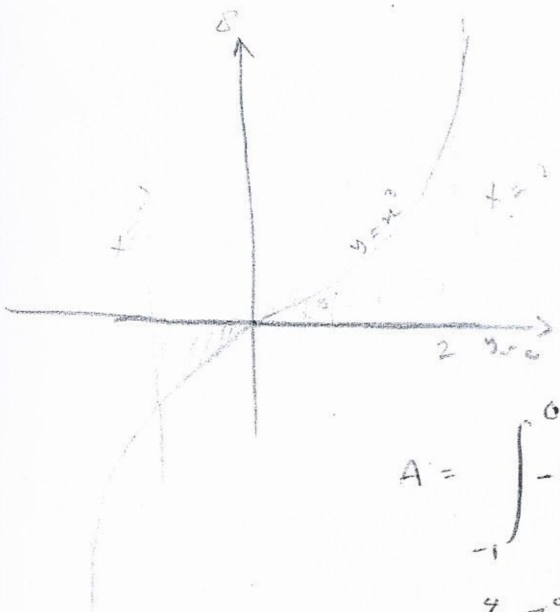
3) $y = x^3$, $y = 0$, $x = -1$, $x = 2$

$y = 0 \implies x^3 = 0$

$y = 1 \implies x = 1$

$y = -1 \Longleftarrow x = -1$

$y = -8 \Longleftarrow x = -2$



$$A = \int_{-1}^{0} -(x^3)\, dx + \int_{0}^{2} x^3\, dx$$

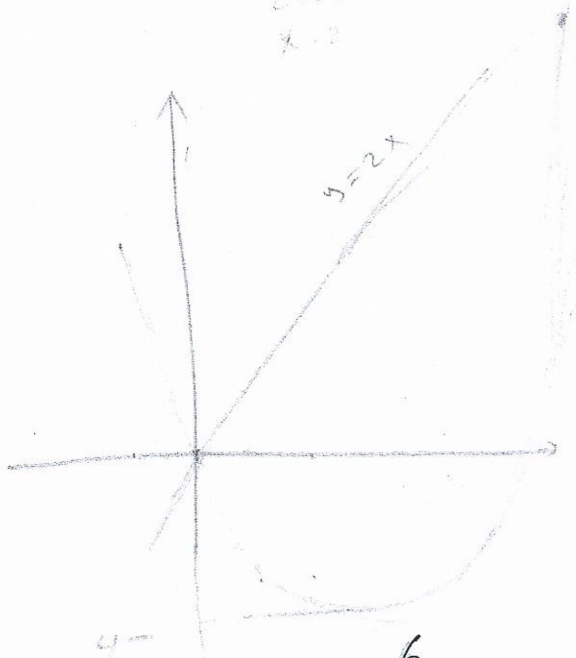
$$= -\frac{x^4}{4}\Bigg]_{-1}^{0} + \left[\frac{x^4}{4}\right]_{0}^{2} = -\left[0 - \frac{1}{4}\right] + \left[\frac{16}{4} - 0\right]$$

$$= \frac{1}{4} + \frac{16}{4} = \frac{17}{4}$$

4) $y = x^2 - 4x \quad , y = 2x$

$2x - 4 = 0$

$x = 2$



$x = -1 \Longrightarrow y = 5$

$y = x^2 - 4x \begin{cases} x = 0 \Longrightarrow y = 0 \\ x = 1 \Longrightarrow y = -3 \\ x = 2 \Longrightarrow y = -4 \end{cases}$

$y = 2x \begin{cases} x = 0 \Longrightarrow y = 0 \\ x = 1 \Longrightarrow y = 2 \end{cases}$

$x^2 - 4x = 2x$

$x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0 \quad , x = 6$

$$A = \int_0^6 (2x-4) - (x^2 - 4x - 4)\, dx$$

$$= x^2 - 4x - \frac{x^3}{3} + 2x^2 + 4x \Big]_0^6 = 3x^2 - \frac{x^3}{3} \Big]_0^6$$

$$= 108 - 72 = 36$$

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5) $y = 2\sqrt{x} \quad , y = 2x - 4 \quad , \quad x = 0$

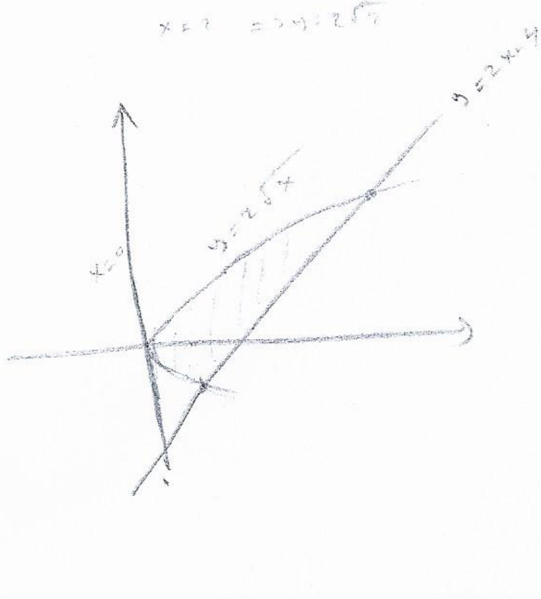
$x = 0 \Longrightarrow y = 0$

$x = 1 \Longrightarrow y = 2$

$x = 2 \Longrightarrow y = 2\sqrt{2}$

$x = 0 \Longrightarrow y = -4$

$x = 1 \Longrightarrow y = -2$



$2\sqrt{x} = 2x - 4$

$4x = 4x^2 - 16x + 16$

$4x^2 - 20x + 16 = 0$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1 \quad , x = 4$

$$A = \int_0^4 (2\sqrt{x} - 2x + 4)\, dx$$

$$= \frac{4}{3} x^{\frac{3}{2}} - x^2 + 4x \Big]_0^4 = \frac{4}{3}(4)^{\frac{3}{2}} - (4)^2 + 4(4)$$

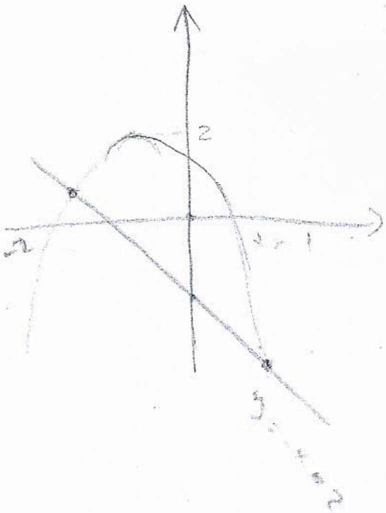
$$= \frac{4}{3}(8) - 16 + 16 = \frac{32}{3}$$

⑥ $x^2 + 2x + y = 0$, $x + y + 2 = 0$

$y = -x^2 - 2x$

$= -(x^2 + 2x)$

$y = -x - 2$



$-x^2 - 2x = x + 2$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x = 1 , x = -2$

$$A = \int_{-2}^{1} (-x^2 - 2x - (-x - 2)\ dx$$

$$= \int_{-2}^{1} (-x^2 - x + 2]\, dx$$

$$= -\frac{x^3}{3} - \frac{x^2}{2} + 2x \Big]_{-2}^{1} = \frac{9}{2}$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \left(-\frac{(-8)}{3} - \frac{(4)}{2} - 4\right)$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \frac{8}{3} + 2 + 4 = 8 - \frac{9}{3} - \frac{1}{2} = \frac{27}{6}$$

⑦ $y = x$, $y = x^3$

$x = x^3$

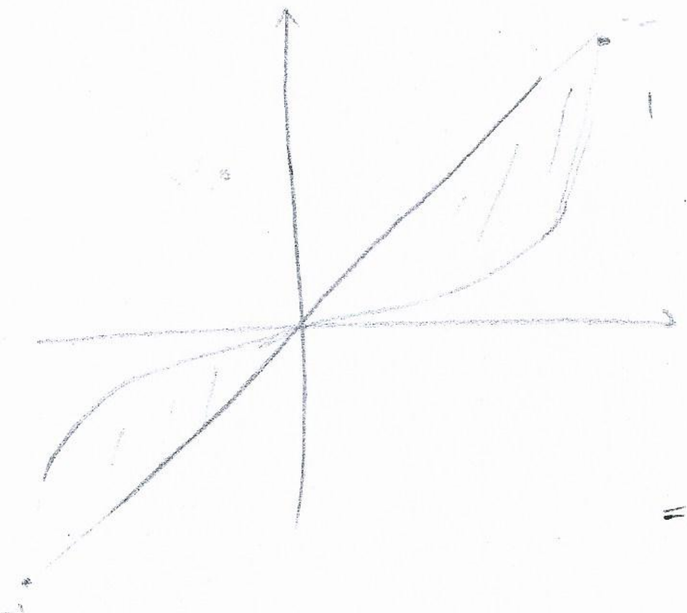
$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0$

$x^2 = 1$

$x = \pm 1$



$$A = \int_{-1}^{0} (x^3 - x)\, dx + \int_{0}^{1} (x - x^3)\, dx$$

$$= \frac{x^4}{4} - \frac{x^2}{2} \Big]_{-1}^{0} + \frac{x^2}{2} - \frac{x^4}{4} \Big]_{0}^{1}$$

$$= 0 - \left[\frac{1}{4} - \frac{1}{2}\right] + \frac{1}{2} - \frac{1}{4} = \frac{-2}{4} + 1$$

$$= -\frac{1}{2} + 1 = \frac{1}{2}$$

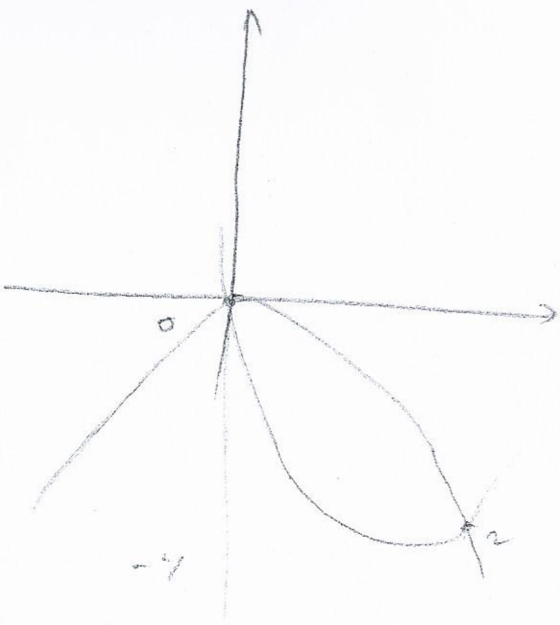
8) $y = x^2 - 4x$ $\quad y = -x^2$

$x^2 - 4x = -x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0 \quad , x = 2$



$$A = \int_0^2 (-x^2 - (x^2 - 4x)\, dx$$

$$= \int_0^2 (-2x^2 + 4x]\, dx$$

$$= \frac{-2x^3}{3} + 2x^2 \Big]_0^2 = -\frac{2}{3}(8) + 2(4)$$

$$= -\frac{16}{3} + 8 = \frac{-16+32}{3}$$

$$= \frac{8}{3}$$

9) $y = \sqrt{x}$ $\quad , y = \frac{x}{2}$

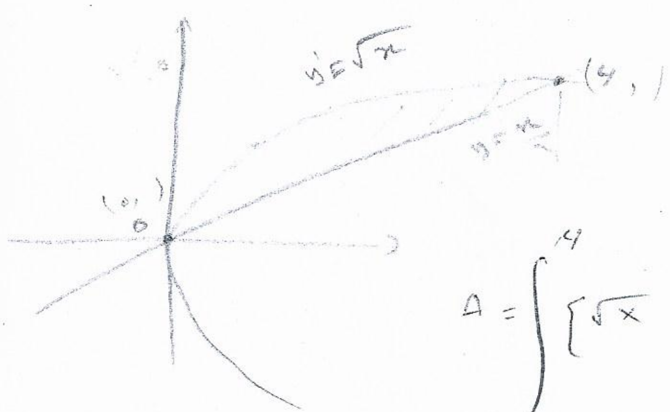
$\sqrt{x} = \frac{x}{2}$

$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad , x = 4$



$$A = \int_0^4 [\sqrt{x} - \frac{x}{2}]\, dx = \frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{4}\Big]_0^4$$

$$= \frac{2}{3}(4)^{\frac{3}{2}} - \frac{16}{4} = \frac{2}{3} \cdot 8 - \frac{16}{4}$$

$$= \frac{16 - 12}{3} = \frac{4}{3}$$

10 $x = 4 - y^2$, $x + y - 2 = 0$

$y^2 = -x + 4$ $y = 2 - x$

$(4, 0)$

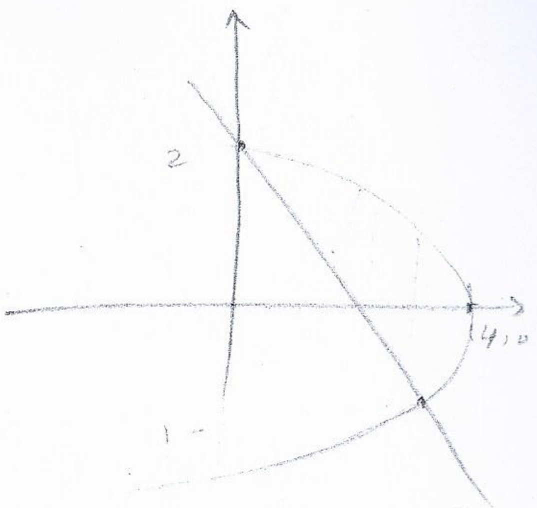
قطع مكافئ، الرأس $(4,0)$

$-x + 4 = 4 - 4x + x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$, $x = 3$



$x = 4 - y^2$

$x = 2 - y$

$-y^2 + 4 = 2 - y$

$-y^2 + y + 4 - 2 = 0$

$-y^2 + y + 2 = 0$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$\boxed{y=-1}$ $\boxed{y=2}$

$$\therefore A = \int_{-1}^{2} (4-y^2) - (2-y)\, dy$$

$$= \int_{-1}^{2} (4 - y^2 - 2 + y)\, dy$$

$$= \int_{-1}^{2} [-y^2 + y + 2]\, dy = -\frac{y^3}{3} + \frac{y^2}{2} + 2y \Big]_{-1}^{2}$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(+\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= -\frac{8}{3} - \frac{1}{3} + \frac{4}{2} - \frac{1}{2} + 4 + 2$$

$$= -\frac{9}{3} + \frac{3}{2} + 6 = \frac{-18 + 9 + 36}{6} = \frac{27}{6} = \frac{9}{2}$$

$y = x^2+1$, $y = 3-x^2$, $X=-2$, $X=2$ 11

$y-1 = x^2$

$(0,1) \rightarrow 1$

$x^2 = -y+3$

$x^2 = -(y-3)$

$(0,3)$

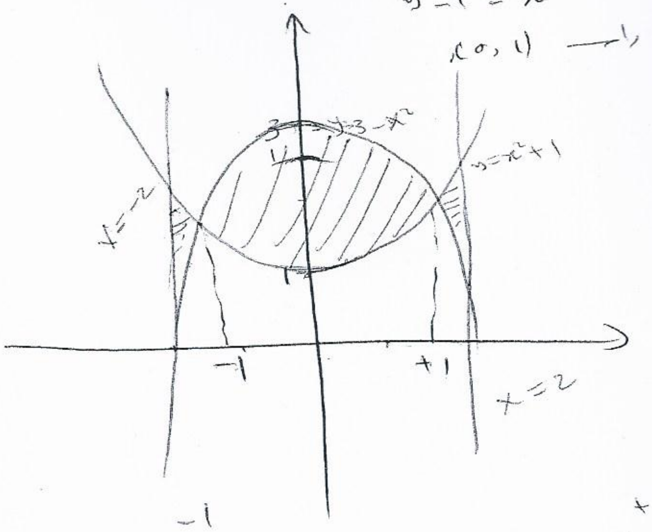
$x^2+1 = 3-x^2$

$2x^2-2=0$

$x^2-1=0$

$x^2=1$

$x=\pm 1$



$$A = \int_{-2}^{-1} (x^2+1) - (3-x^2)\,dx + \int_{-1}^{+1} ((3-x^2)-(x^2+1)\,dx + \int_{1}^{2} [x^2+1-3+x^2]\,dx$$

$$= \int_{-2}^{-1} [2x^2-2]\,dx + \int_{-1}^{1} (2)\,dx + \int_{1}^{2} [2x^2-2]\,dx$$

$$= \frac{2}{3}x^3 - 2x\Big]_{-2}^{-1} + 2x\Big]_{-1}^{1} + \frac{2}{3}x^3 - 2x\Big]_{1}^{2}$$

$$= \frac{2}{3}(-1) - 2(-1) - \left[\frac{2}{3}(-8) - 2(-2)\right] + 2 - (-2) + \frac{2}{3}(8) - 4 - \left[\frac{2}{3} - 2\right]$$

$$= -\frac{2}{3} + 2 + \frac{16}{3} + 4 + 2 + 2 + \frac{16}{3} - 4 - \frac{2}{3} + 2$$

$$= -\frac{4}{3} + \frac{32}{3} + 8 = \frac{28}{3} + 8 = \frac{52}{3}$$

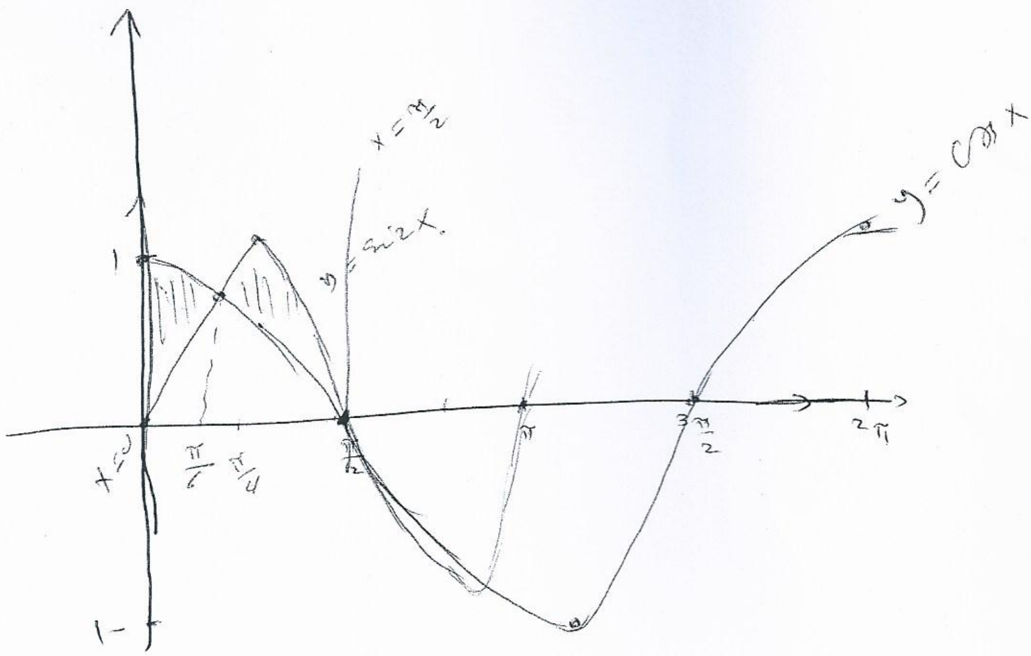
$y = \cos x \;,\; y = \sin 2x \;,\; x = 0,\; x = \frac{\pi}{2}$ 12

$\cos x = \sin 2x$

$\cos\frac{\pi}{6} = \sin 2\frac{\pi}{6}$

$\frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$



$$A = \int_0^{\frac{\pi}{6}} [\cos x - \sin 2x]\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [\sin 2x - \cos x]\,dx$$

$$= \sin x + \frac{1}{2}\cos 2x \Big]_0^{\frac{\pi}{6}} + \Big[-\frac{1}{2}\cos 2x + \sin x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \sin x + \frac{1}{2}\cos 2x\Big]_0^{\frac{\pi}{6}} - \Big(\frac{1}{2}\cos 2x - \sin x$$

$$= \sin\frac{\pi}{6} + \frac{1}{2}\cos\frac{\pi}{3} - \Big(\sin 0 + \frac{1}{2}\cos 0\Big) - \Big[\frac{1}{2}\cos\pi - \sin\frac{\pi}{2} - \Big(\frac{1}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\Big)\Big]$$

$$= \frac{1}{2} + \frac{1}{2}\Big(\frac{1}{2}\Big) - \frac{1}{2} - \Big[-\frac{1}{2} - \Big(\frac{1}{2}\Big)\Big(\frac{1}{2}\Big) - \frac{1}{2}\Big]$$

$$= \cancel{\frac{1}{2}} + \frac{1}{4} - \cancel{\frac{1}{2}} - \Big(-\cancel{\frac{1}{2}} - \frac{1}{4} + \cancel{\frac{1}{2}}\Big)$$

$$= \cancel{\frac{2}{2} - \frac{1}{4} + \frac{3}{2} + \frac{1}{4} = 2}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$x = y^4$, $x = 2 - y^4$

$y^4 = 2 - y^4$

$2y^4 - 2 = 0$

$y^4 - 1 = 0$

$(y^2)^2 - 1 = 0$

$(y^2 - 1)(y^2 + 1) = 0$

$\boxed{y = \pm 1}$ $\quad y = \pm i$

$$A = \int_{-1}^{1} (2 - y^4) - (y^4)\, dy$$

$$= 2y - 2\frac{y^5}{5}\Big]_{-1}^{1} = 2 - \frac{2}{5} - \left[-2 + \frac{2}{5}\right]$$

$$= 4 - \frac{4}{5} = \frac{20 - 4}{5} = \frac{16}{5}$$

س٣/ احسب قيمة كلاً من :

1) $\int_0^2 |x^2 - x^3| \, dx$

ايجاد صفر المقياس

$x^2 - x^3 = 0$

$x^2(1-x) = 0$

$x^2 = 0$ or $1 - x = 0$

$\boxed{x = 0}$ $\boxed{x = 1}$

$$\begin{array}{ccccc} + & 0 & + & 1 & - \end{array}$$

$$A = \int_0^1 (x^2 - x^3)\,dx + \int_1^2 (x^3 - x^2)\,dx$$

$$= \frac{x^3}{3} - \frac{x^4}{4}\Big]_0^1 + \Big[\frac{x^4}{4} - \frac{x^3}{3}\Big]_1^2$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{16}{4} - \frac{8}{3} - \left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{16}{4} - \frac{8}{3} - \frac{1}{4} + \frac{1}{3} = \frac{-6}{3} + \frac{14}{4}$$

$$= \frac{-24 + 42}{12} = \frac{18}{12} = \frac{3}{2}$$

(3) $\int_0^{\pi} \left| \sin x - \frac{2}{\pi} x \right| dx$

$\sin x - \frac{2}{\pi} x = 0$

$x = \frac{\pi}{2}$

$\sin \frac{\pi}{2} - \frac{2}{\pi}\left(\frac{\pi}{2}\right) = 1 - 1 = 0$

$$A = \int_0^{\frac{\pi}{2}} \left[\sin x - \frac{2}{\pi} x\right] dx + \int_{\frac{\pi}{2}}^{\pi} -\left(\sin x - \frac{2}{\pi} x\right) dx$$

$$= -\cos x - \frac{2}{\pi}\frac{x^2}{2} \Big]_0^{\frac{\pi}{2}} - \left[-\cos x - \frac{2}{\pi}\frac{x^2}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\cos\frac{\pi}{2} - \frac{\left(\frac{\pi}{2}\right)^2}{\pi} - \left(-\cos 0 - \frac{0}{\pi}\right) - \left[-\cos\pi - \frac{\pi^2}{\pi} - \left(-\cos\frac{\pi}{2} - \frac{\left(\frac{\pi}{2}\right)^2}{\pi}\right)\right]$$

$$= 0 - \frac{\pi^2}{4}\cdot\frac{1}{\pi} + 1 - \left[1 - \pi - \left(0 - \frac{\pi^2}{4}\cdot\frac{1}{\pi}\right)\right]$$

$$= -\frac{\pi}{4} + \cancel{1} - \cancel{1} + \pi - \frac{\pi}{4} = -\frac{2\pi}{4} + \pi = \frac{-2\pi + 4\pi}{4}$$

$$= \frac{2\pi}{4} = \frac{1}{2}\pi$$