(5.2) ـــ تمرينا

Find the volume of the solid with cross-sectional area $A(x)$:

(1) $A(x) = x + 2 \quad , \quad -1 \leq x \leq 3$

$$V = \int_{-1}^{3} (x+2)\,dx = \frac{x^2}{2} + 2x \Big]_{-1}^{3} = \frac{9}{2} + 6 - \left(\frac{1}{2} - 2\right)$$

$$= \frac{9+12}{2} + \frac{3}{2} = \frac{24}{2} = 12$$

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(3) $A(x) = \pi(4-x)^2 \quad , 0 \leq x \leq 2$

$$V = \pi \int_{0}^{2} (4-x)^2\,dx = -\pi \int_{0}^{2} -(4-x)^2\,dx$$ دالة x مشتقتها

$$= -\frac{\pi}{3}(4-x)^3\Big]_{0}^{2} = -\frac{\pi}{3}\left[(4-2)^3 - (4-0)^3\right]$$

$$= -\frac{\pi}{3}[8 - 64] = \frac{56}{3}\pi$$

Compute the volume of the solid formed by revolving the given region about the given line.

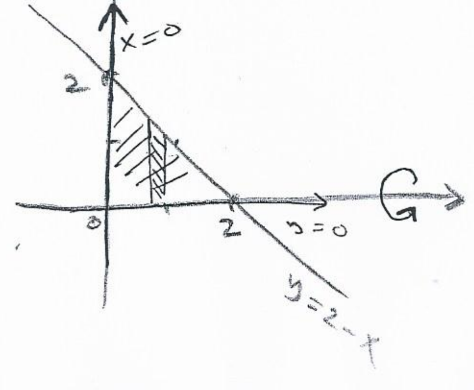
(17) Region bounded by $y = 2-x$, $y = 0$ and $x = 0$ about ~~(a)~~

(a) the x-axis

(Disk)

(b) $y = 3$

محور الدوران داخل منطقة التكامل



$y = 2 - x \quad , y = 0$

$2 - x = 0 \Rightarrow x = 2$

$y = 2 - x \quad , \quad x = 0$

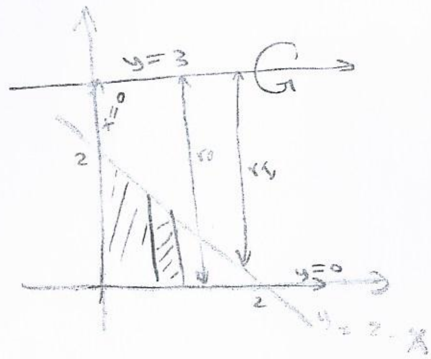
$y = 2 - 0 \Rightarrow y = 2$

(a) $V = \pi \int_0^2 (2-x)^2\,dx = -\pi \int_0^2 -(2-x)^2\,dx$

$$= -\frac{\pi}{3}\left[(2-x)^3\right]_0^2 = -\frac{\pi}{3}\left[(2-2)^3 - (2-0)^3\right]$$

$$= -\frac{\pi}{3}\left[0 - 8\right) = \frac{8\pi}{3}$$

(b) (washers)



$$V = \pi \int_0^2 \left[r_o^2 - r_I^2\right] dx$$

$r_o = 3-0 = 3$

$r_I = 3-(2-x) = 3-2+x$
$= 1+x$

$$V = \pi \int_0^2 \left[(3)^2 - (1+x)^2\right] dx = \pi \left[\int_0^2 [9]\,dx - \int_0^2 [1+x]^2\,dx\right]$$

$$= \pi \left[9x\Big|_0^2 - \frac{(1+x)^3}{3}\Big|_0^2\right] = \pi\left[9(2) - \left[\frac{(1+2)^3}{3} - \left[\frac{(1+0)^3}{3}\right]\right]\right]$$

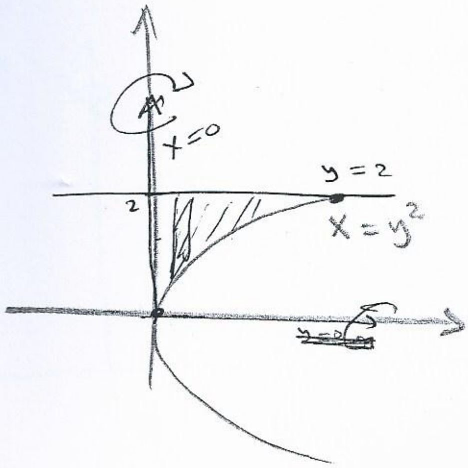
$$= \pi\left[18 - \left(\frac{27}{3} - \frac{1}{3}\right)\right] = \pi\left[18 - \left(\frac{26}{3}\right)\right] = \pi\left[\frac{54-26}{3}\right]$$

$$= \frac{28}{3}\pi$$

(19) Region bounded by $y=\sqrt{x}$, $y=2$ and $x=0$ about

(a) the y-axies

(b) $x=4$

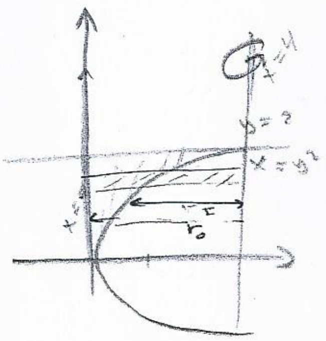


(a)

$$V=\pi\int_0^2 (y^2)^2 dy$$

$$=\pi\int_0^2 y^4 dy = \pi \frac{y^5}{5}\Big|_0^2 = \frac{32}{5}\pi$$

(b) $x=4$



$$\cancel{V=\pi\int_0^2 [4]^2 dy}$$

$$V=\pi\int_0^2 \left[(r_o)^2 - r_I^2\right] dy$$

$$V=\pi\int_0^2 \left[(4)^2-(4-y^2)^2\right] dy$$

$$=\pi\left[\int_0^2 [16]\,dy - \int_0^2 (4-y^2)^2 dy\right]$$

$$=\pi\left[16y\Big|_0^2 - \int_0^2 (16-8y^2+y^4)\,dy\right]$$

$$=\pi\left[16(2-0) - \left(16y-\frac{8}{3}y^3+\frac{y^5}{5}\Big|_0^2\right)\right]$$

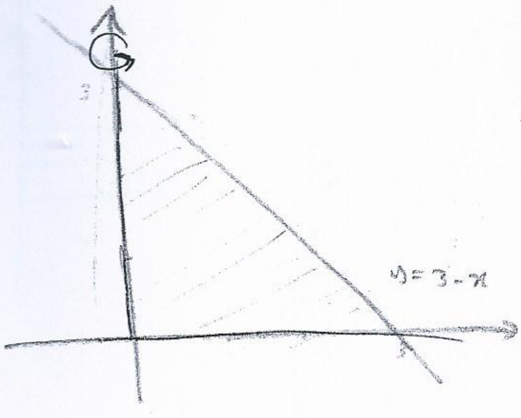
$$=\pi\left[32-\left(16(2)-\frac{8}{3}(2)^3+\frac{(2)^5}{5}\right)\right]$$

$$=\pi\left[\cancel{32}-\cancel{32}+\frac{64}{3}-\frac{32}{5}\right] = \pi\left[\frac{320-96}{15}\right]$$

$$=\pi\left[\frac{224}{15}\right]$$

(25) Let R the region bounded by $y=3-x$, the x-axis and the y-axis. Compute the volume of the solid formed by revolving R about the given line.

a) the y-axis



(a) $V = \pi \int_0^3 (3-y)^2 \, dy$

$= -\pi \int_0^3 -(3-y)^2 \, dy$

$= -\pi \left[\frac{(3-y)^3}{3} \right]_0^3 = -\pi \left[\frac{(3-3)^3 - (3-0)^3}{3} \right]$

$= -\pi \left[0 - \frac{27}{3} \right]$

$= 9\pi$

b) the x-axis

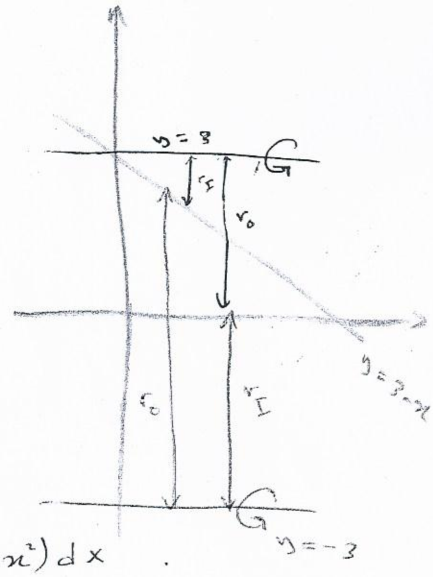
(b) $V = \pi \int_0^3 (3-x)^2 \, dx$

$= -\pi \left[\frac{(3-x)^3}{3} \right]_0^3$

$= -\frac{\pi}{3} \left[(3-3)^3 - (3-0)^3 \right]$

$= 9\pi$

(c) $y = 3$



(c) $V = \pi \int_0^3 \left[(3)^2 - \left(3 - (3-x) \right)^2 \right] dx$

$= \pi \int_0^3 \left[9 - x^2 \right] dx = \pi \left[9x - \frac{x^3}{3} \right]_0^3$

$= \pi \left[27 - \frac{27}{3} - 0 \right] = 18\pi$

(d) $y = -3$

(d) $V = \pi \int_0^3 \left[((3-x)+3)^2 - (3)^2 \right] dx$

$= \pi \int_0^3 \left[(6-x)^2 - 9 \right] dx = \pi \int_0^3 (27 - 12x + x^2) \, dx$

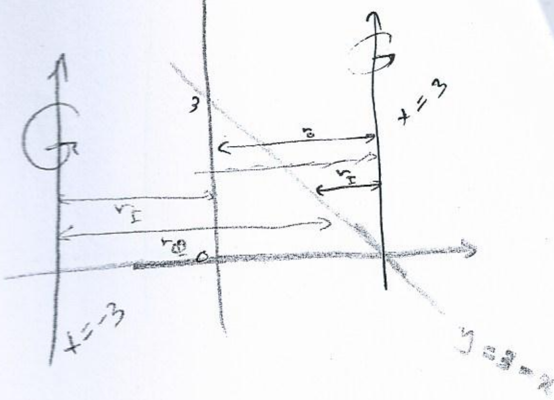
$= \pi \left(27x - 6x^2 + \frac{x^3}{3} \right]_0^3 = \pi \left[27(3) - 6(9) + \frac{27}{3} - 0 \right]$

$= 36\pi$

(e) $x=3$

(e) $V = \pi \int_0^3 [(3)^2 - (3-(3-y))^2]\,dy$

$= \pi \int_0^3 [9 - y^2]\,dy = \pi \left[9y - \frac{y^3}{3}\right]_0^3$



$= \pi \left[27 - \frac{27}{3}\right] = 18\pi$

(f) $x=-3$

(f) $V = \pi \int_0^3 [((3-y)+3)^2 - (3)^2]\,dy$

$= \pi \int_0^3 [(6-y)^2 - 9]\,dy$

$= \pi \int_0^3 [27 - 12y + y^2]\,dy = \pi \left[27y - 6y^2 + \frac{y^3}{3}\right]_0^3$

$= 36\pi$

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(27) Let R be the region bounded by $y=x^2$, $y=0$ and $x=1$. Compute the volume of the solid formed by revolving R about the given line.

(a) the y-axis

$y=x^2$, $x=1$

$y=1$

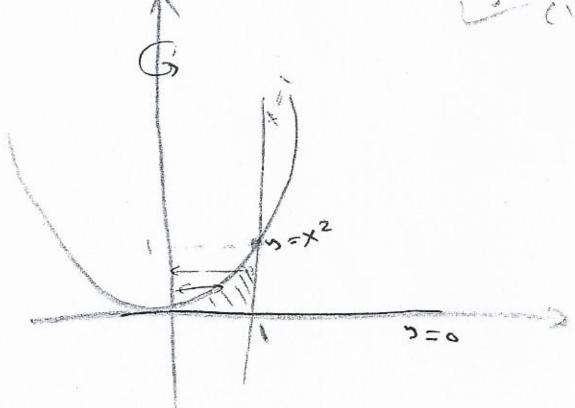
$(1,1)$

(a)

$V = \int_0^1 \pi (1)^2\,dy - \int_0^1 \pi (\sqrt{y})^2\,dy$

$= \pi \int_0^1 (1-y)\,dy = \pi \left[y - \frac{y^2}{2}\right]_0^1$

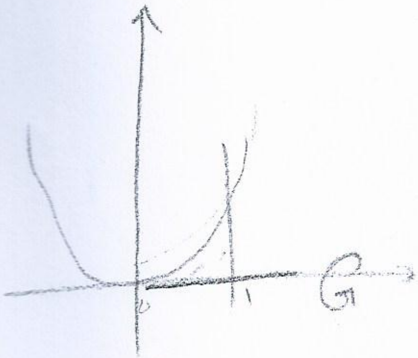
$= \frac{\pi}{2}$



(b) the x - axis

b) $V = \pi \int_0^1 (x^2)^2 \, dx$

$= \pi \frac{x^5}{5}\Big|_0^1 = \frac{\pi}{5}$



c) $\boxed{X=1}$

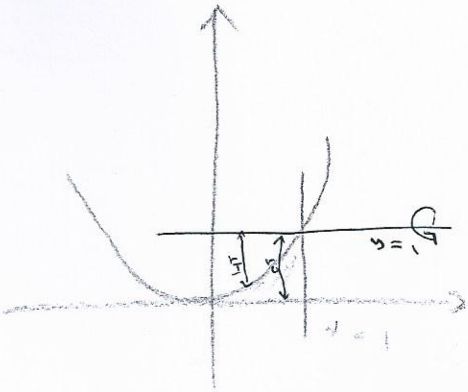
$v = \pi \int_0^1 (1-\sqrt{y})^2 \, dy$

$= \pi \int_0^1 (1 - 2\sqrt{y} + y) \, dy$

$= \pi \left(y - \frac{4}{3} y^{\frac{3}{2}} + \frac{y^2}{2}\right)\Big|_0^1$

$= \frac{\pi}{6}$

d) $\boxed{y=1}$



$V = \pi \int_0^1 \left[(1)^2 - (1-x^2)^2\right] dx$

$= \pi \int_0^1 \left[1 - (1 - 2x^2 + x^4)\right] dx$

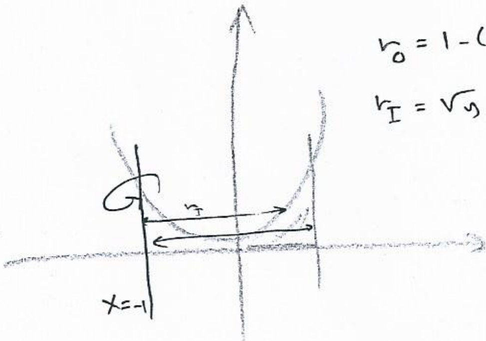
$= \pi \int_0^1 \left[2x^2 - x^4\right] dx = \pi \left[\frac{2}{3}x^3 - \frac{x^5}{5}\right]_0^1$

$= \pi \left[\frac{2}{3} - \frac{1}{5}\right] = \frac{7}{15}\pi$

e) $\boxed{x=-1}$

$r_o = 1-(-1) = 2$

$r_I = \sqrt{y} - (-1)$



$V = \pi \int_0^1 \left[(2)^2 - (1+\sqrt{y})^2\right] dy$

$= \pi \int_0^1 \left[4 - (1 + 2\sqrt{y} + y)\right] dy$

$= \pi \int_0^1 \left[3 - 2y^{\frac{1}{2}} - y\right] dy = \pi \left[3y - \frac{4}{3}y^{\frac{3}{2}} - \frac{y^2}{2}\right]_0^1$

$= \frac{7}{6}\pi$

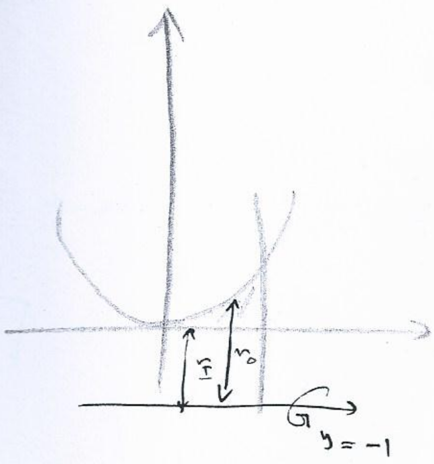
(f) $\boxed{y = -1}$

$$V = \pi \int_0^1 \left[(x^2-(-1))^2 - (0-(-1)) \right] dx$$

$$= \pi \int_0^1 \left[(x^2+1)^2 - (1) \right] dx$$

$$= \pi \int_0^1 \left[x^4 + 2x^2 \cancel{+1} \cancel{-1} \right] dy$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} \right]_0^1 = \frac{13}{15}\pi$$



Compute the volume :-

1) The region bounded by $y = x^2$ and the x-axis , $-1 \le x \le 1$ revolved about $x = 2$

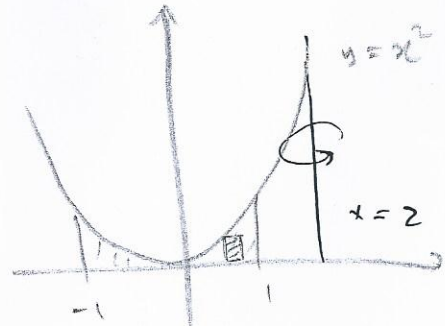
$\boxed{\text{shell}}$

$$V = \int_{-1}^{1} 2\pi \overset{\text{البعد}}{(2 - x)} \overset{\text{الارتفاع}}{(x^2)} \, dx$$

$$= 2\pi \int_{-1}^{1} (2x^2 - x^3)\, dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1} = 2\pi \left[\left(\frac{2}{3} - \frac{1}{4}\right) - \left(\frac{-2}{3} - \frac{1}{4}\right) \right]$$

$$= 2\pi \left[\frac{4}{3} \right] = \frac{8\pi}{3}$$



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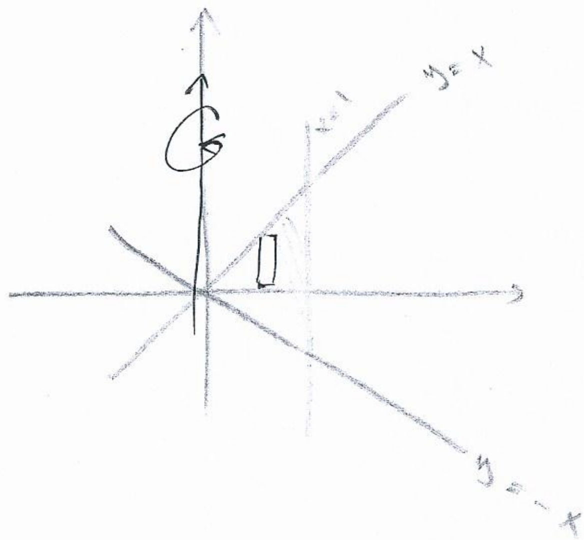
3) The region bounded by $y = x$, $y = -x$ and $x = 1$ revolved about the y-axis

$$V = \int_{0}^{1} \left[2\pi (x - 0)(x - (-x)) \right] dx$$

$$= \int_{0}^{1} 2\pi x (2x) \, dx$$

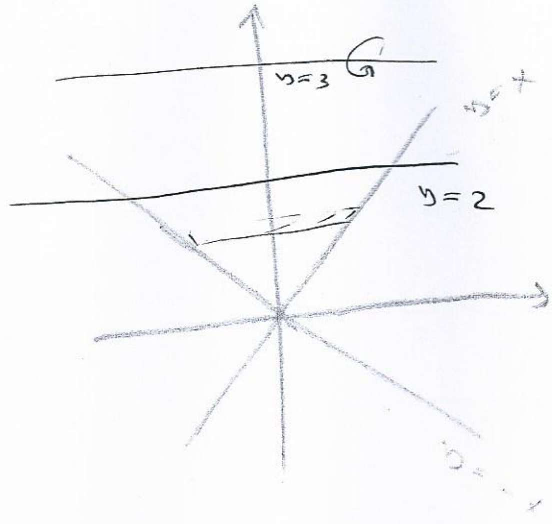
$$= 4\pi \int_{0}^{1} x^2 \, dx = 4\pi \left. \frac{x^3}{3} \right|_{0}^{1}$$

$$= \frac{4\pi}{3}$$



(5) region bounded by $y=x$, $y=-x$ and $y=2$ revolved

about $y=3$



$$V=\int_0^2 \left[2\pi(3-y)(y-(-y))\right]dy$$

$$=2\pi\int_0^2 (6y-2y^2)\,dy$$

$$=2\pi\left[3y^2-\frac{2}{3}y^3\right]_0^2$$

$$=2\pi\left[12-\frac{2}{3}(8)\right]=2\pi\left[12-\frac{16}{3}\right]$$

$$=\frac{40}{3}\pi$$