

Q3.

$$y = \cos^{-1} 2x + \sin^{-1} \left(\frac{1}{x^2} \right)$$

$$\cos^{-1} x = \frac{1}{\sqrt{x^2-1}}$$

$$\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$y' = \left[2 \cdot \frac{1}{\sqrt{4x^2-1}} \right] + \left[\frac{-2}{x^3} \cdot \frac{1}{\sqrt{1-\frac{1}{x^2}}} \right]$$

$$= \frac{2}{\sqrt{4x^2-1}} - \frac{2}{x^3 \sqrt{1-x^{-2}}}$$

Q4.

$$y = x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = (1 + \ln x) \cdot y$$

$$= (1 + \ln x) x^x$$

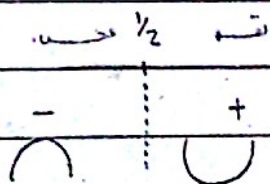
Q5.

$$y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$$

$$y' = x^2 - x - 2 \Rightarrow y'' = 2x - 1$$

$$y'' = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

نقطة انحناء



Concave up: $(\frac{1}{2}, \infty)$

Concave down: $(-\infty, \frac{1}{2})$

بالنقطة
عاشق التمدد

Q1. Choose the correct result

1. $\int 3x \cot x^2 dx$

☐ A $-\frac{3}{2} \ln |\csc x^2 + \cot x^2| + C$

☐ C $\frac{3}{2} \ln |\sec x^2| + C$

☐ B $\frac{3}{2} \ln |\sec x^2 + \tan x^2| + C$

☐ D $\frac{3}{2} \ln |\sin x^2| + C$

1mark

2. $\int \sinh 7x dx$

☐ A $\cosh (7x) + C$

☐ C $7 \cosh (7x) + C$

☐ B $\frac{1}{7} \cosh (7x) + C$

☐ D $\frac{1}{7} \sinh (7x) + C$

1mark

3. $\int \frac{1}{x^2 - 1} dx$

☐ A $\tan^{-1} x + C$

☐ C $\cot^{-1} x + C$

☐ B $\frac{1}{2} \ln \frac{x+1}{x-1} + C$

☐ D $\frac{1}{2} \ln \frac{x-1}{x+1} + C$

1mark

The derivative of y with respect to x for $y = 4 \cosh\left(\frac{x}{5}\right)$ is

☐ A $4 \sinh\left(\frac{x}{5}\right)$

☐ C $-4 \sinh\left(\frac{x}{5}\right)$

☐ B $-\frac{4}{5} \sinh\left(\frac{x}{5}\right)$

☐ D $\frac{4}{5} \sinh\left(\frac{x}{5}\right)$

1mark

12: Evaluate

1. $\int \frac{x^3 dx}{x^4 - 2}$

2marks

2. $\int \frac{5x - 3}{x^2 - 2x - 3} dx$

2marks

3. $\int \frac{e^{5/x}}{x^2} dx$

2marks

4. $\int \tan^{-1} 2x dx$

2marks

Q1. ①

$$\int 3x \cdot \cot x^2 \, dx$$

D ∞

$$= \frac{3}{2} \int \frac{\cos x^2}{\sin x^2} \, dx$$

$$= \frac{3}{2} \ln |\sin x^2| + C$$

②

$$\left(\frac{1}{7}\right) \int \sinh 7x \, dx$$

B ∞

$$= \frac{1}{7} \cosh 7x + C$$

③

$$- \int \frac{1}{x^2 - 1} \, dx$$

$$= \int \frac{1}{(x-1)(x+1)} \, dx = \int \frac{A}{x-1} \, dx + \int \frac{B}{x+1} \, dx$$

D ∞

$$\star = A \ln(x-1) + B \ln(x+1) + C$$

A, B ايجاد

$$A(x+1) + B(x-1) = 1$$

$$Ax + A + Bx - B = 1$$

$$\textcircled{1} \quad A + B = 0$$

$$\textcircled{2} \quad A - B = 1 \Rightarrow 2A = 1 \rightarrow A = \frac{1}{2}$$

$$\textcircled{1} \quad A + B = 0 \rightarrow \frac{1}{2} \ln(x-1) - \frac{1}{2} \ln(x+1) + C$$

$$B = -\frac{1}{2}$$

$$\frac{1}{2} [\ln(x-1) - \ln(x+1)] + C$$

$$\div (1 \leftarrow (-) \text{ في } \ln) \quad \frac{1}{2} \ln \frac{x-1}{x+1} + C$$

④ The derivative $\rightarrow y = 4 \cosh\left(\frac{x}{5}\right)$

$$y = 4 \sinh\left(\frac{x}{5}\right) \cdot \left(\frac{1}{5}\right) \rightarrow \frac{4}{5} \sinh\left(\frac{x}{5}\right)$$

$$= \frac{4}{5} \sinh\left(\frac{x}{5}\right) + C$$

100

Q2. ①

$$\frac{1}{4} \int \frac{x^3}{x^4 - 2} dx$$

$$= \frac{1}{4} \ln |x^4 - 2| + C$$

②

$$\int \frac{5x - 3}{x^2 - 2x - 3} dx$$

$$= \frac{5x - 3}{(x - 3)(x + 1)} = \frac{A}{x - 3} + \frac{B}{x + 1}$$

$$= A \ln |x - 3| + B \ln |x + 1| + C$$

أى، A و B

$$A(x + 1) + B(x - 3) = 5x - 3$$

$$Ax + A + Bx - 3B = 5x - 3$$

$$\text{①} \rightarrow \begin{cases} A + B = 5 \end{cases} \rightarrow A = 5 - B$$

$$\text{②} \rightarrow \begin{cases} A - 3B = -3 \end{cases}$$

$$\text{③} \rightarrow \begin{cases} 5 - B - 3B = -3 \end{cases} \rightarrow \boxed{B = 2}$$

التعويض بـ B في A

$$\boxed{A} = 5 - B = 5 - 2 = \boxed{3}$$

$$= 3 \ln |x - 3| + 2 \ln |x + 1| + C$$

$$(3) \int \frac{e^{\frac{s}{x}}}{x^2} dx$$

بالتعويض $u = \frac{s}{x} \Rightarrow du = -\frac{1}{x^2} dx$

$$= \int e^u \cdot (-1) du$$

$$= -\frac{1}{-1} e^u + C$$

$$(4) \int \tan^{-1} 2x dx$$

التعويض $u = \tan^{-1} 2x \Rightarrow du = \frac{1}{1+4x^2} dx$

$$du = 1 \Rightarrow v = x$$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$= x \tan^{-1} 2x - \int \frac{x}{1+4x^2} dx$$

$$= x \tan^{-1} 2x - \frac{1}{8} \ln |1+4x^2| + C$$

التوفيق للجميع

مشاركتي لكم