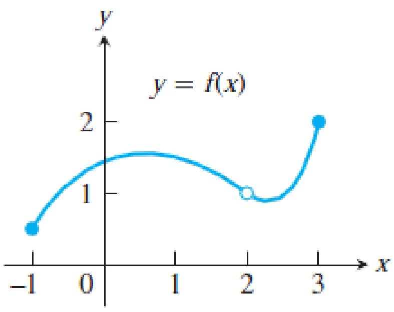
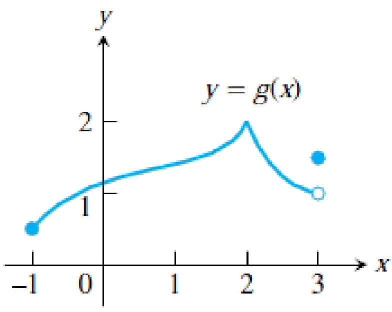
Exercises of Continuity

In Exercises 1–4, say whether the function graphed is continuous on $[-1, 3]$. If not, where does it fail to be continuous and why?

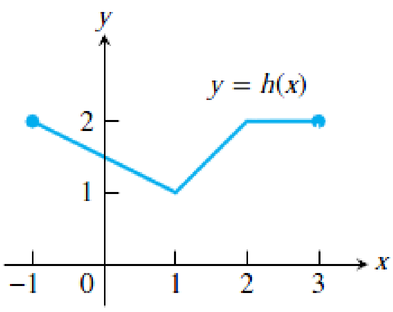
1.



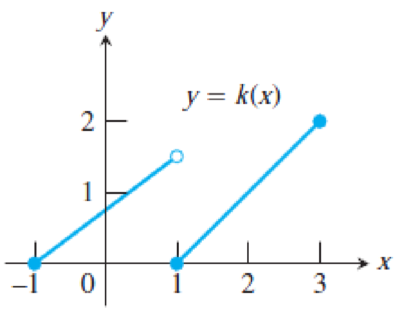
2.



3.



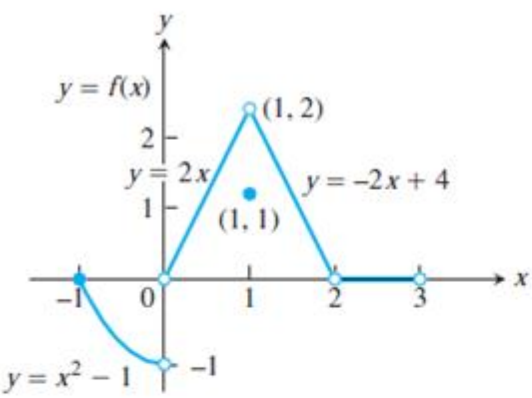
4.



Exercises 5–10 are about the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$

graphed in the accompanying figure.



The graph for Exercises 5–10.

5. **a.** Does $f(-1)$ exist?
   b. Does $\lim_{x \to -1^+} f(x)$ exist?
   c. Does $\lim_{x \to -1^+} f(x) = f(-1)$?
   d. Is f continuous at $x = -1$?
6. **a.** Does $f(1)$ exist?
   b. Does $\lim_{x \to 1} f(x)$ exist?
   c. Does $\lim_{x \to 1} f(x) = f(1)$?
   d. Is f continuous at $x = 1$?
7. **a.** Is f defined at $x = 2$? (Look at the definition of f.)
   b. Is f continuous at $x = 2$?
8. At what values of x is f continuous?
9. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?
10. To what new value should $f(1)$ be changed to remove the discontinuity?

At what points are the functions in Exercises 13–28 continuous?

14. $y = \dfrac{1}{(x+2)^2} + 4$

16. $y = \dfrac{x+3}{x^2 - 3x - 10}$

18. $y = \dfrac{1}{|x| + 1} - \dfrac{x^2}{2}$

20. $y = \dfrac{x+2}{\cos x}$

22. $y = \tan \dfrac{\pi x}{2}$

24. $y = \dfrac{\sqrt{x^4 + 1}}{1 + \sin^2 x}$

26. $y = \sqrt[4]{3x - 1}$

28. $y = (2 - x)^{1/5}$

Find the limits in Exercises 29–34. Are the functions continuous at the point being approached?

29. $\lim_{x \to \pi} \sin (x - \sin x)$

30. $\lim_{t \to 0} \sin \left(\frac{\pi}{2} \cos (\tan t) \right)$

31. $\lim_{y \to 1} \sec (y \sec^2 y - \tan^2 y - 1)$

32. $\lim_{x \to 0} \tan \left(\frac{\pi}{4} \cos (\sin x^{1/3}) \right)$

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35. Define $g(3)$ in a way that extends $g(x) = (x^2 - 9)/(x - 3)$ to be continuous at $x = 3$.

36. Define $h(2)$ in a way that extends $h(t) = (t^2 + 3t - 10)/(t - 2)$ to be continuous at $t = 2$.

37. Define $f(1)$ in a way that extends $f(s) = (s^3 - 1)/(s^2 - 1)$ to be continuous at $s = 1$.

38. Define $g(4)$ in a way that extends $g(x) = (x^2 - 16)/(x^2 - 3x - 4)$ to be continuous at $x = 4$.

39. For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x?

40. For what value of b is

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

continuous at every x?

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49. Solving an equation If $f(x) = x^3 - 8x + 10$, show that there are values c for which $f(c)$ equals **(a)** π; **(b)** $-\sqrt{3}$; **(c)** 5,000,000.