Exercises of Limits 1

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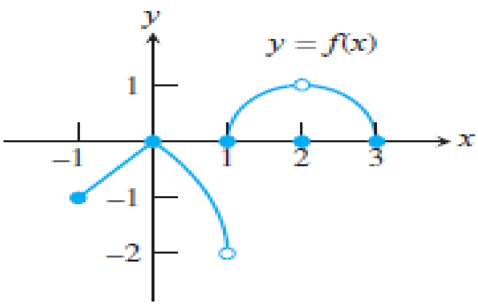
4. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?
   a. $\lim_{x \to 2} f(x)$ does not exist.
   b. $\lim_{x \to 2} f(x) = 2$.
   c. $\lim_{x \to 1} f(x)$ does not exist.
   d. $\lim_{x \to x_0} f(x)$ exists at every point x_0 in $(-1, 1)$.
   e. $\lim_{x \to x_0} f(x)$ exists at every point x_0 in $(1, 3)$.



Find the limits

21. $\lim_{x \to 2} 2x$

23. $\lim_{x \to 1/3} (3x - 1)$

25. $\lim_{x \to -1} 3x(2x - 1)$

27. $\lim_{x \to \pi/2} x \sin x$

13. $\lim_{y \to -3} (5 - y)^{4/3}$

15. $\lim_{h \to 0} \dfrac{3}{\sqrt{3h + 1} + 1}$

17. $\lim_{h \to 0} \dfrac{\sqrt{3h + 1} - 1}{h}$

19. $\lim_{x\to 5} \frac{x-5}{x^2-25}$

21. $\lim_{x\to -5} \frac{x^2+3x-10}{x+5}$

23. $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1}$

25. $\lim_{x\to -2} \frac{-2x-4}{x^3+2x^2}$

27. $\lim_{u\to 1} \frac{u^4-1}{u^3-1}$

29. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

31. $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$

33. $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

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49. If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$.

Prove the limit statements

37. $\lim_{x\to 4} (9-x) = 5$

39. $\lim_{x\to 9} \sqrt{x-5} = 2$

41. $\lim_{x\to 1} f(x) = 1$ if $f(x) = \begin{cases} x^2, & x \ne 1 \\ 2, & x = 1 \end{cases}$

Find the limits

17. a. $\lim_{x\to -2^+} (x+3)\frac{|x+2|}{x+2}$ b. $\lim_{x\to -2^-} (x+3)\frac{|x+2|}{x+2}$

14. $\lim_{x \to 1^-} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right)$

15. $\lim_{h \to 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$

24. $\lim_{h \to 0^-} \frac{h}{\sin 3h}$

27. $\lim_{x \to 0} \frac{x \csc 2x}{\cos 5x}$

34. $\lim_{x \to 0} \frac{\sin 5x}{\sin 4x}$

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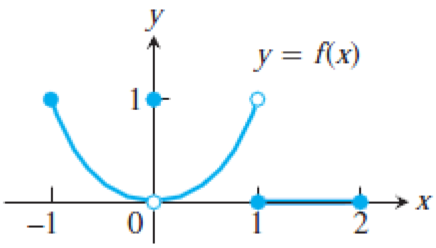
1. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



a. $\lim_{x \to -1^+} f(x) = 1$

b. $\lim_{x \to 0^-} f(x) = 0$

c. $\lim_{x \to 0^-} f(x) = 1$

d. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

e. $\lim_{x \to 0} f(x)$ exists

f. $\lim_{x \to 0} f(x) = 0$

g. $\lim_{x \to 0} f(x) = 1$

h. $\lim_{x \to 1} f(x) = 1$

i. $\lim_{x \to 1} f(x) = 0$

j. $\lim_{x \to 2^-} f(x) = 2$

k. $\lim_{x \to -1^-} f(x)$ does not exist.

l. $\lim_{x \to 2^+} f(x) = 0$

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find the limit of each rational function **(a)** as $x \to \infty$ and **(b)** as $x \to -\infty$.

47. $f(x) = \dfrac{2x + 3}{5x + 7}$

49. $f(x) = \dfrac{x + 1}{x^2 + 3}$

51. $h(x) = \dfrac{7x^3}{x^3 - 3x^2 + 6x}$

53. $g(x) = \dfrac{10x^5 + x^4 + 31}{x^6}$

54. $h(x) = \dfrac{9x^4 + x}{2x^4 + 5x^2 - x + 6}$

Find the limits

17. $\lim \dfrac{1}{x^2 - 4}$ as

a. $x \to 2^+$ **b.** $x \to 2^-$

c. $x \to -2^+$ **d.** $x \to -2^-$

20. $\lim \dfrac{x^2 - 1}{2x + 4}$ as

a. $x \to -2^+$ **b.** $x \to -2^-$

c. $x \to 1^+$ **d.** $x \to 0^-$

Find the equations of the asymptotes

27. $y = \dfrac{1}{x - 1}$

29. $y = \dfrac{1}{2x + 4}$

31. $y = \dfrac{x + 3}{x + 2}$

33. $y = \dfrac{x^2}{x - 1}$

35. $y = \dfrac{x^2 - 4}{x - 1}$

37. $y = \dfrac{x^2 - 1}{x}$