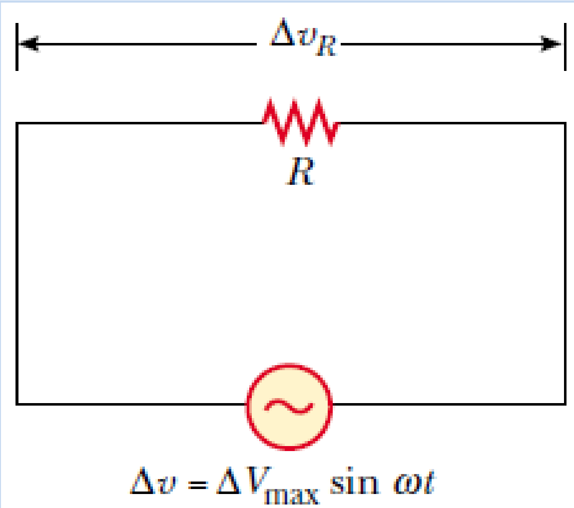
المقاومة في دوائر التيار المتردد Resistors in an AC Circuit



بتطبيق قانون كيرشوف للجهود للدائرة المغلقة:

$$\Delta v + \Delta v_R = 0$$

$$\Delta v - i_R R = 0$$

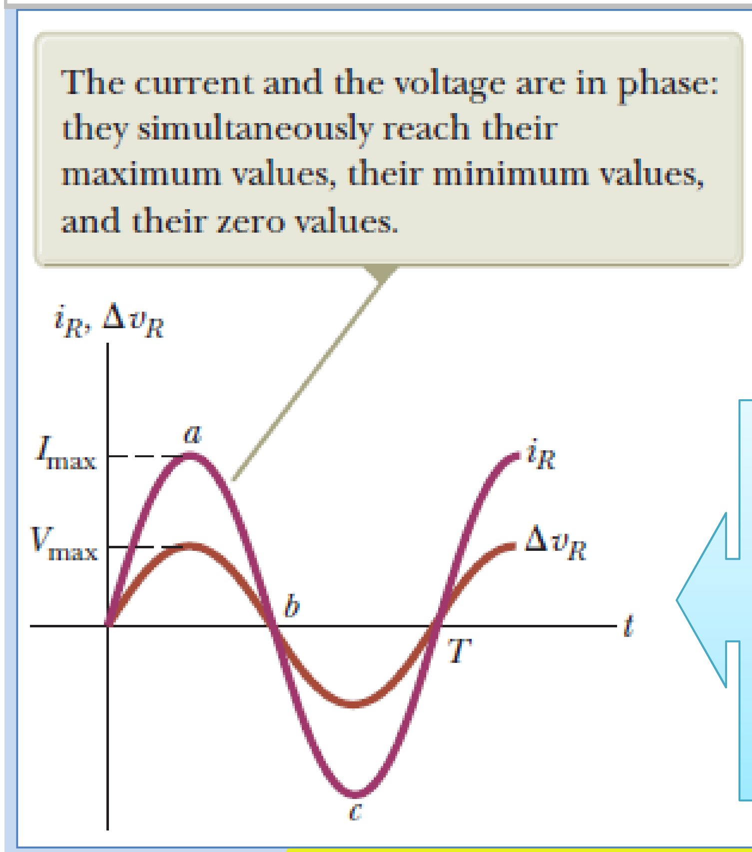
$$\Delta v = i_R R = \Delta V_{max} \sin \omega t$$

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

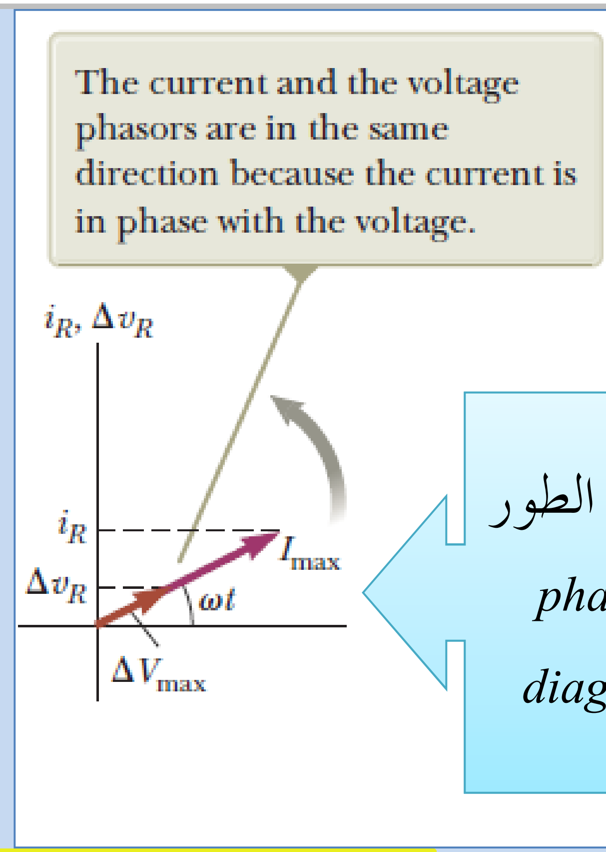
الجهد على طرفي المقاومة (من المعادلة السابقة):

$$\Delta v_R = i_R R = I_{max} R \sin \omega t$$

Resistors in an AC Circuit المقاومة في دوائر التيار المتردد



تغير الجهد و التيار خلال المقاومة مع الزمن.

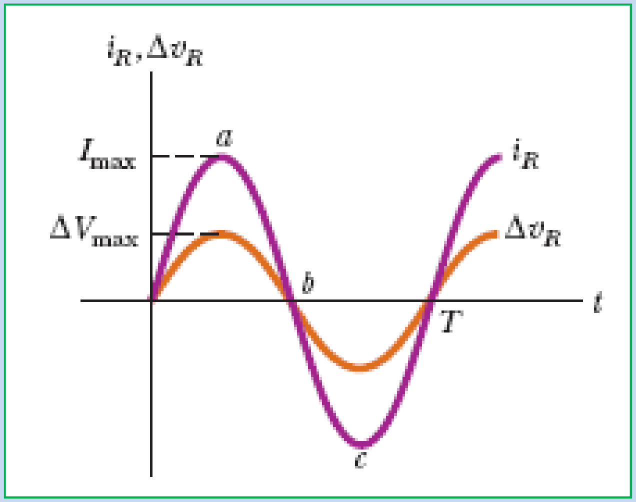


مخطط الطور *phasor diagram*

الجهد و التيار يتغيران بنفس الطريقة مع الزمن (لهما نفس الطور).

A phasor is a vector whose length is proportional to the maximum value of the variable it represents, and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable. The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

المقاومة في دوائر التيار المتردد Resistors in an AC Circuit



نتيجة

يصل التيار و الجهد عند القيمة العظمى عند نفس الزمن أي لهما نفس الطور (in phase).

- اتجاه التيار لا يؤثر على المقاومة في الدائرة.
- تُنتج التصادمات بين الالكترونات و ذرات المقاومة زيادة في درجة حرارة المقاومة بغض النظر عن اتجاه التيار.
- يوصف معدل تحول الطاقة الكهربائية إلى طاقة داخلية بالقدرة.

Resistors behave essentially the same way in both DC and AC circuits

القدرة

$$P = i^2 R$$

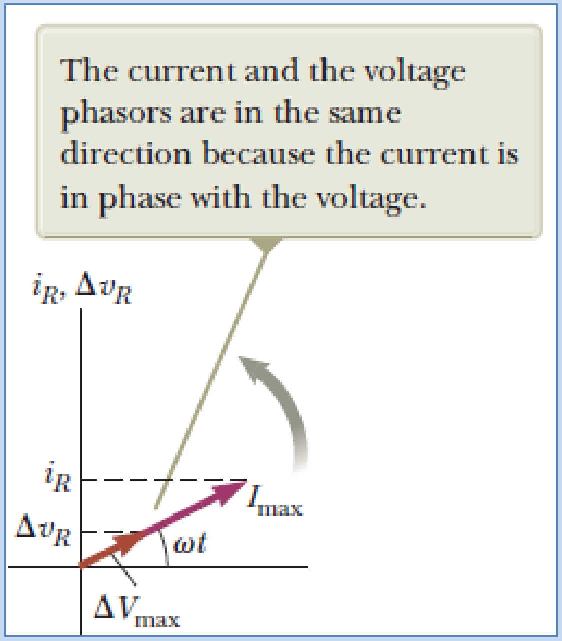
التيار اللحظي

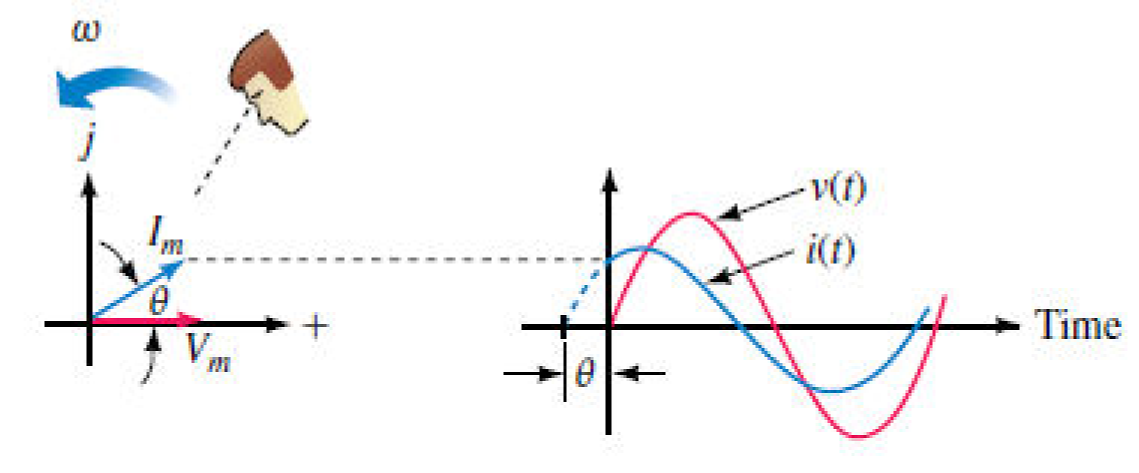
توضيح

NOTES...

1. Although we have indicated phasor rotation in Figure 15–35 by a series of "snapshots," this is too cumbersome; in practice, we show only the phasor at its $t = 0$ s (reference) position and imply rotation rather than show it explicitly.

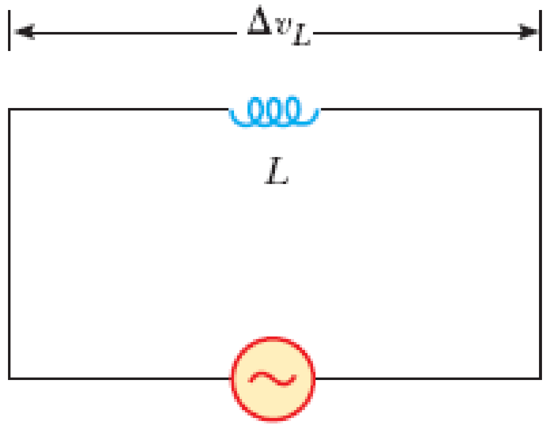
2. Although we are using maximum values (E_m and I_m) here, phasors are normally drawn in terms of effective values (considered in Section 15.9). For the moment, we will continue to use maximum values. We make the change in Chapter 16.





(a) I_m leads V_m (b) Therefore, $i(t)$ leads $v(t)$

Inductors in an AC Circuit الملف في دوائر التيار المتردد



If $\Delta v_L = \varepsilon_L = -L(di/dt)$ is the self-induced instantaneous voltage across the inductor, then Kirchhoff's loop rule applied to this circuit gives:

$$\Delta v + \Delta v_L = 0$$

$$\Delta v - L\frac{di_L}{dt} = 0$$

1

$$\Delta v = \Delta V_{max} \sin \omega t = L\frac{di_L}{dt}$$

$$di_L = \frac{\Delta V_{max}}{L} \sin \omega t \, dt$$

بالتكامل للحصول على التيار المار في الملف كدالة في الزمن

$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t \, dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

Inductors in an AC Circuit الملف في دوائر التيار المتردد

$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t \, dt = -\frac{\Delta V_{\max}}{\omega L}\cos \omega t$$

$$\cos \omega t = -\sin(\omega t - \pi/2)$$

بالتعويض بالمفكوك السابق ينتج:

2

$$i_L = \frac{\Delta V_{\max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

1

$$\Delta v = \Delta V_{\max} \sin \omega t = L\frac{di_L}{dt}$$

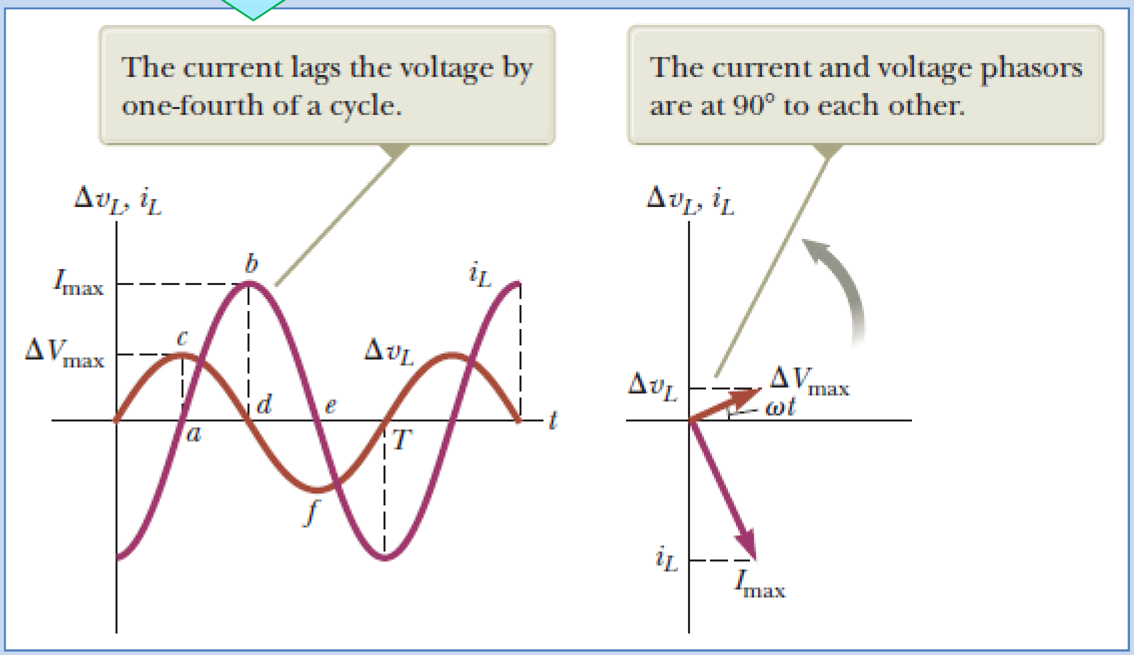
نتيجة

Comparing Equations 1 and 2, we see that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L *across the inductor are out of phase* by ($\pi/2$) rad = 90°.

Inductors in an AC Circuit الملف في دوائر التيار المتردد

the current in an inductor always *lags behind the voltage* across the inductor by 90° (one-quarter cycle in time).

تغير الجهد و التيار خلال الملف مع الزمن



Inductors in an AC Circuit الملف في دوائر التيار المتردد

$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t\, dt = -\frac{\Delta V_{max}}{\omega L}\cos \omega t$$

متى تحصل أقصى قيمة للتيار المار في الملف

$$\cos \omega t = \pm 1$$

$$I_{max} = \frac{\Delta V_{max}}{\omega L}$$

بمقارنتها بمعادلة التيار المستمر I=ΔV/R ومطابقة الوحدات ينتج أن ωL تقاس بوحدة **الأوم**.

The expression for the rms current in an inductor is similar to Equation 33.9, with I_{max} replaced by I_{rms} and ΔV_{max} replaced by ΔV_{rms}.

It must behave in a manner similar to resistance. Notice that because ωL *depends on the applied frequency ω, the* inductor *reacts differently, in terms of offering resistance to current, for different* frequencies. For this reason, we define ωL *as the* ***inductive reactance:***

تُعَرف المقاومة الفعلية للملف في دوائر التيار المتردد بالمفاعلة الحثية X_L وتساوي:

$$X_L = \omega L = 2\pi f L$$

معامل الحث الذاتي / هنري

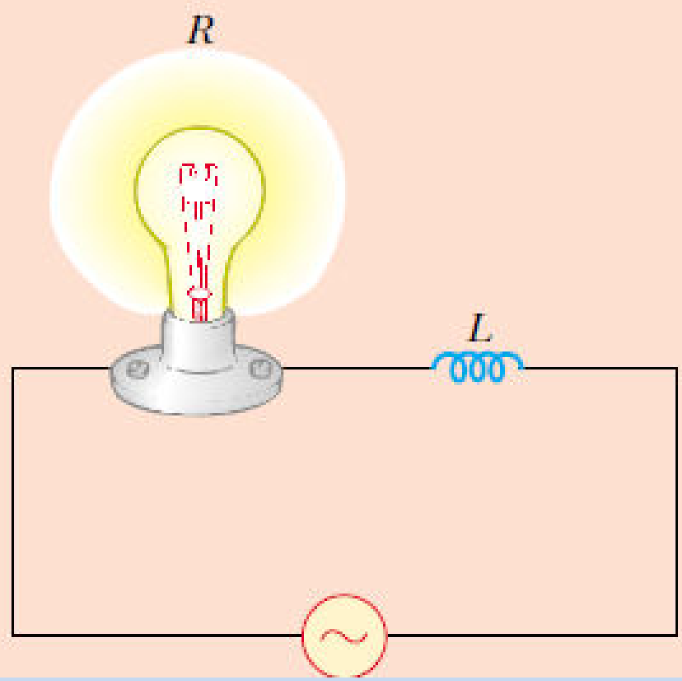
$$X_L = \omega L = 2\pi\ f\ L$$

X_L تعتمد على L لأن ق د ك الخلفية تزداد بزيادة L. وكذلك على f لأن ق د ك الخلفية تعتمد أيضاً على $\Delta I/\Delta t$ وتكون كبيرة إذا كان التغير في التيار سريعاً و التردد عالياً.

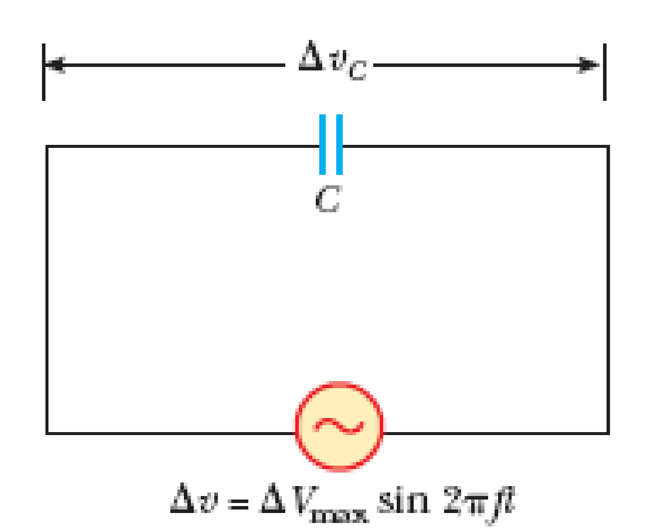
الجهد عبر طرفي الملف يصبح:

$$\Delta v_L = -L\frac{di_L}{dt} = -\Delta V_{max} \sin \omega t = -I_{max} X_L \sin \omega t$$

Quick Quiz 33.4 Consider the AC circuit in Figure 33.8. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at (a) high frequencies (b) low frequencies (c) The brightness will be the same at all frequencies.



Capacitors in an AC Circuit المكثف في دوائر التيار المتردد



بتطبيق قانون كيرشوف للجهود للدائرة المغلقة:

$$\Delta v - \frac{q}{C} = 0$$

Δv : $\Delta V_{max} \sin\omega t$

$\frac{q}{C}$: ΔV_C

$$q = C\Delta V_{max} \sin \omega t$$

لإيجاد التيار اللحظي للدائرة نفاضل الشحنة بالنسبة للزمن فينتج:

$$i_C = \frac{dq}{dt} = \omega C\Delta V_{max} \cos \omega t$$

باستخدام المفكوك

$$\cos \omega t = \sin(\omega t + \frac{\pi}{2})$$

$$i_C = \omega C\Delta V_{max} \sin(\omega t + \frac{\pi}{2})$$

المكثف في دوائر التيار المتردد Capacitors in an AC Circuit

معادلة الجهد

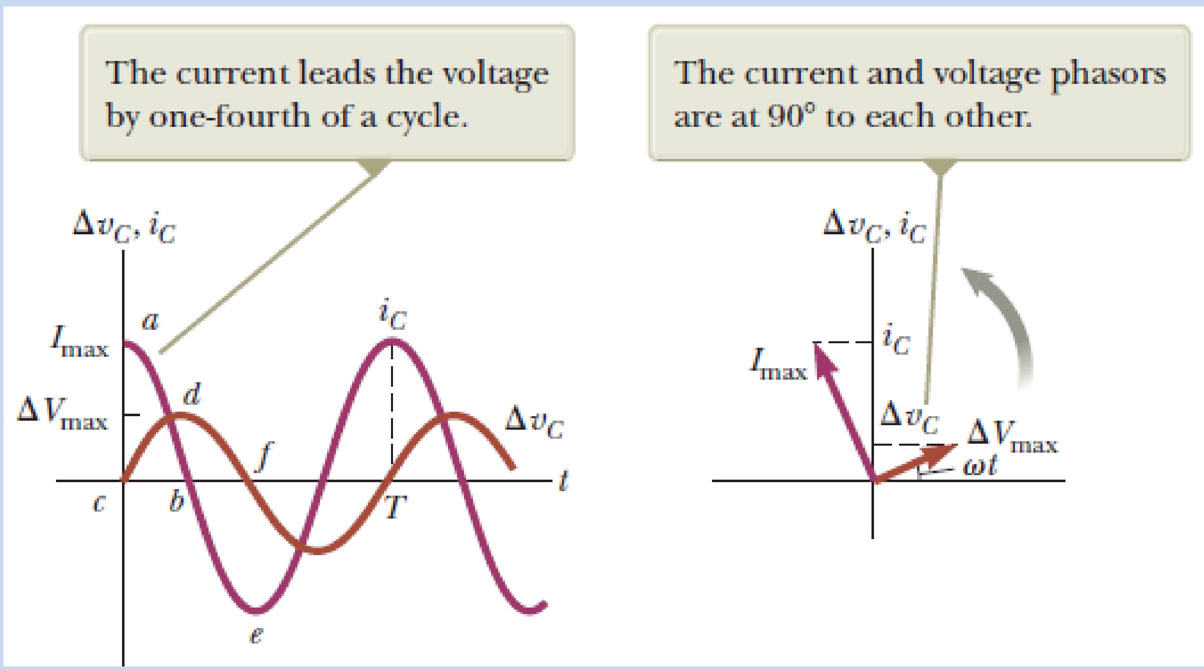
$$\Delta v = \Delta V_{max} \sin \omega t$$

معادلة التيار

$$i_C = \omega C \Delta V_{max} \sin(\omega t + \frac{\pi}{2})$$

نتيجة

يصل التيار و الجهد عند القيمة العظمى و بينهما فرق في الطور مقدار 90^o.



المكثف في دوائر التيار المتردد Capacitors in an AC Circuit

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

أقصى قيمة للتيار عند
$\cos\omega t = \pm 1$

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{1/\omega C}$$

بمقارنتها بمعادلة التيار المستمر
ومطابقة $I=\Delta V/R$
الوحدات ينتج أن $(1/\omega L)$
تقاس بوحدة **الأوم**.

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$

المفاعلة السعوية capacitive reactance

$$X_C = \frac{1}{2\pi f C})$$

سعة المكثف بالفاراد

التردد بالهيرتز

التردد الزاوي ω

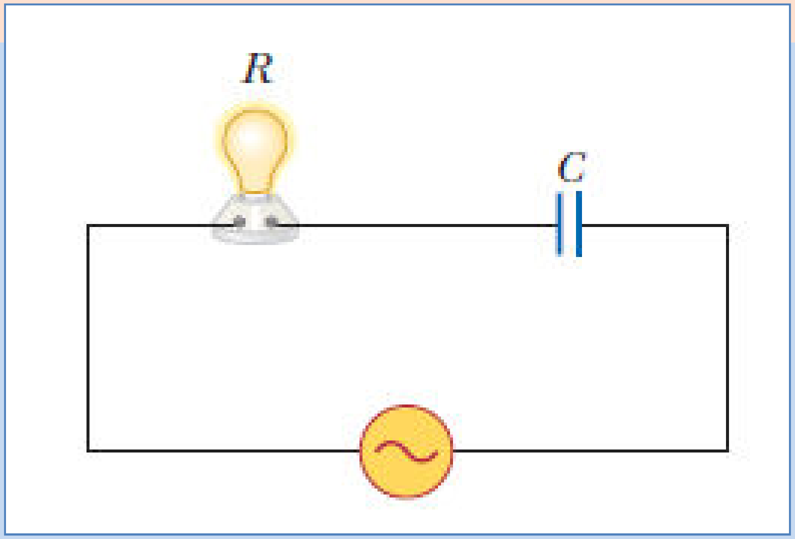
المكثف في دوائر التيار المتردد Capacitors in an AC Circuit

يكتب الجهد عبر المكثف بالعلاقة:

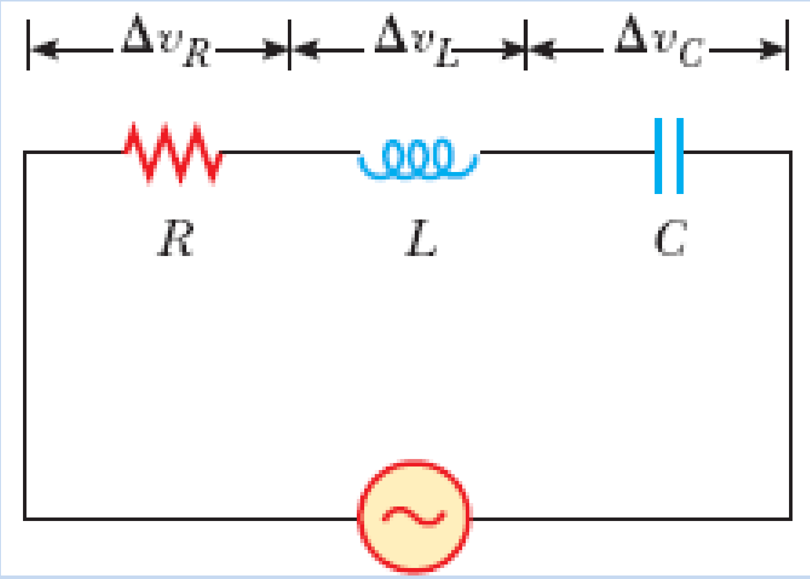
$$\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$$

و تكتب القيم الفعالة للتيار و الجهد بنفس العلاقات السابقة باستبدال Imax و Vmax بـــ Irms و Vrms.

Quick Quiz 33.5 Consider the AC circuit in Figure 33.11. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at (a) high frequencies (b) low frequencies (c) The brightness will be same at all frequencies.



The RLC series Circuit دائرة RLC توالي



ماذا يحدث عند توصيل R , L , C في دائرة على التوالي ؟

$$i = I_{\max} \sin 2\pi f t$$

the applied voltage varies sinusoidally with time. It is convenient to assume that the instantaneous applied voltage is given by

$$\Delta v = \Delta V_{\max} \sin \omega t$$

while the current varies as

$$i = I_{\max} \sin(\omega t - \phi)$$

First, we note that because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, the current at all points in a series AC circuit has the same amplitude and phase.

ماذا عن الجهود ؟

The RLC series Circuit دائرة RLC توالي

الجهد عبر المقاومة في نفس الطور مع التيار

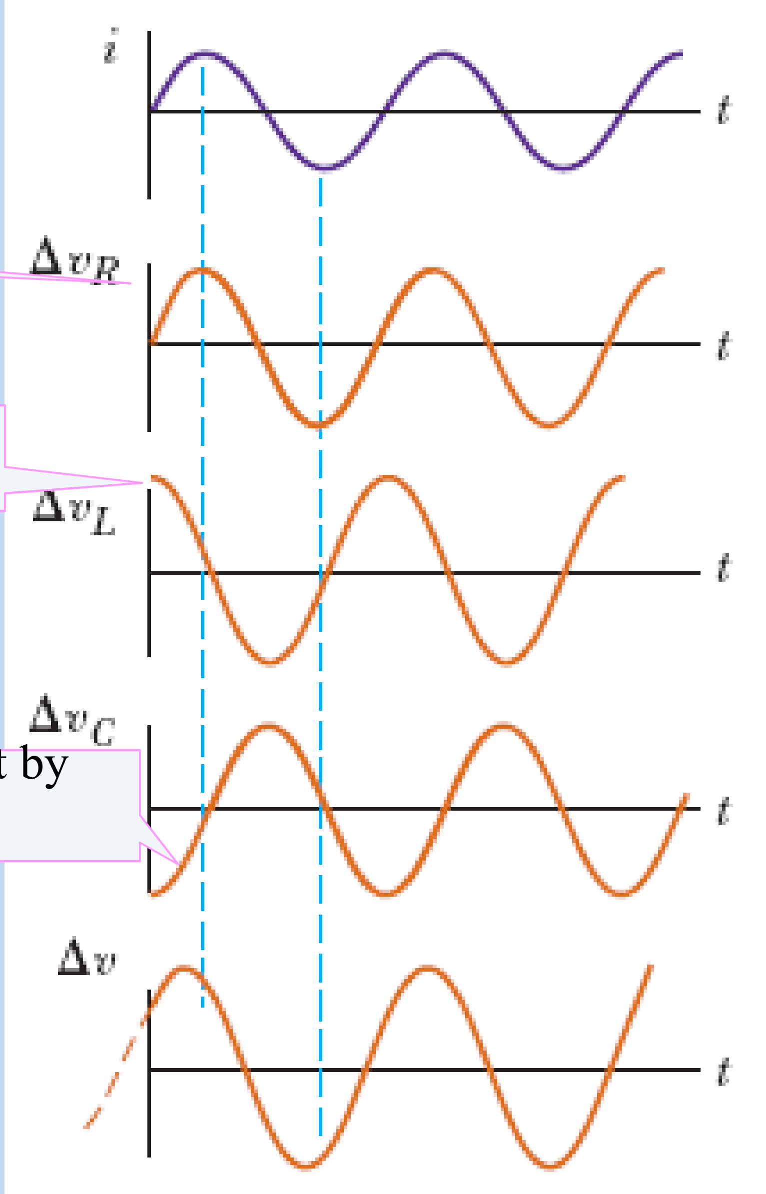
$$\Delta v_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t$$

the voltage across the inductor leads the current by 90°

$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

the voltage across the capacitor lags behind the current by 90°

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$



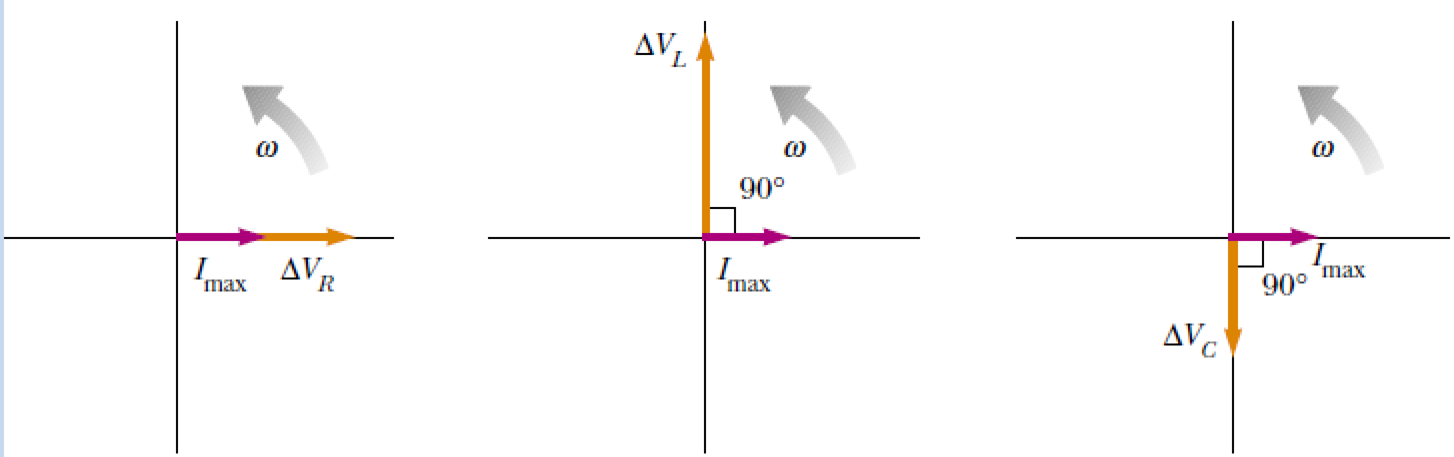
دائرة RLC توالي The RLC series Circuit

قيمة الجهد اللحظي Δv للمصدر المتردد يساوي:

$$\Delta v = \Delta v_R + \Delta v_C + \Delta v_L$$

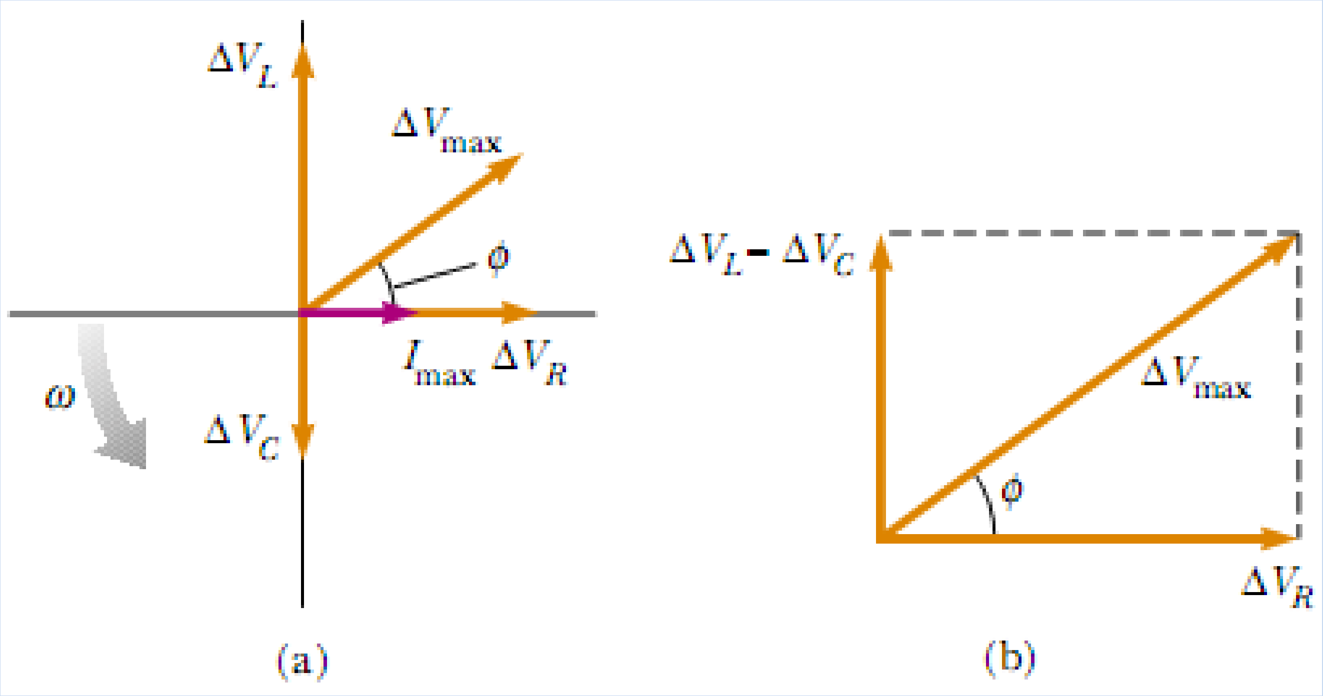
هذا لا يعني أنه يمكن جمع الجهود المقاسة عبر R, L, C بواسطة الفولتميتر (ac) لتعطي جهد المصدر.

لماذا؟



(a) Resistor (b) Inductor (c) Capacitor

The RLC series Circuit دائرة RLC توالي

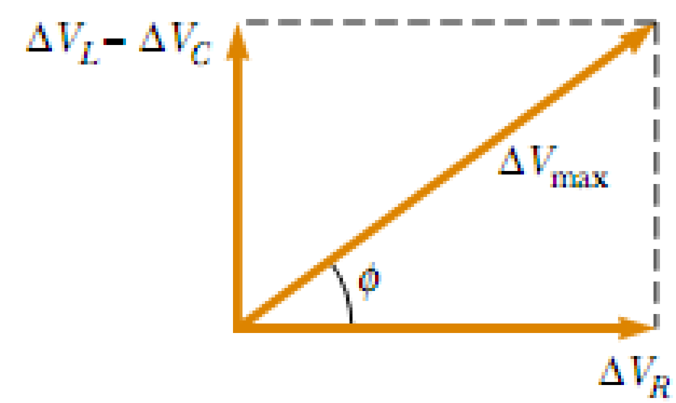


(a) The phasor ΔV_R is in phase with the current phasor I_{max}, ΔV_L leads I_{max} by 90°, and ΔV_C lags I_{max} by 90°.

The total voltage ΔV_{max} makes an angle ϕ with I_{max}.

(b) Simplified version of the phasor diagram.

The RLC series Circuit دائرة RLC توالي



$$\Delta V_{max} = \sqrt{\Delta V_R^{\,2} + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + (X_L - X_C)^2} \qquad (33.24)$$

Therefore, we can express the maximum current as

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Maximum current in an *RLC* circuit

Once again, this has the same mathematical form as Equation 27.8. The denominator of the fraction plays the role of resistance and is called the **impedance** Z of the circuit:

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \qquad (33.25)$$

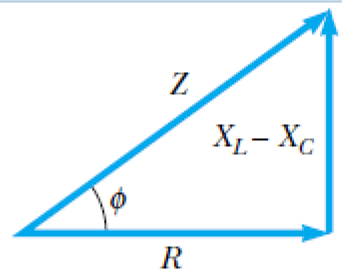
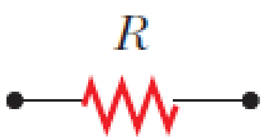
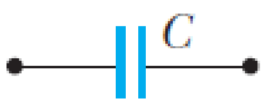
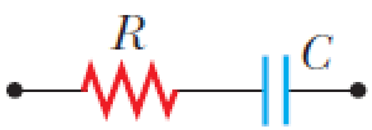
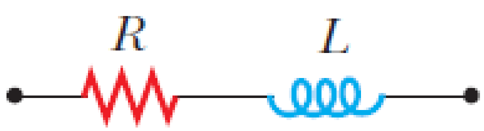
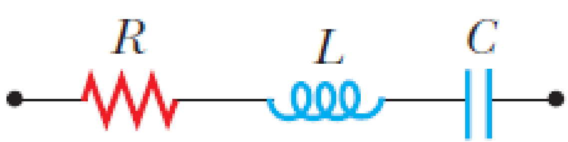


Figure 33.16 An impedance triangle for a series *RLC* circuit gives the relationship $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Impedance Values and Phase Angles for Various Combinations of Circuit Elements[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Resonance in a Series RLC Circuit الرنين في دوائر RLC توالي

قيمة I_{rms} في دائرة RLC

$$I_{rms} = \frac{\Delta V_{rms}}{Z} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

يصل التيار إلى قيمته العظمى عند أقل قيمة للمعاوقة، وذلك عند:

$$X_L = X_C$$

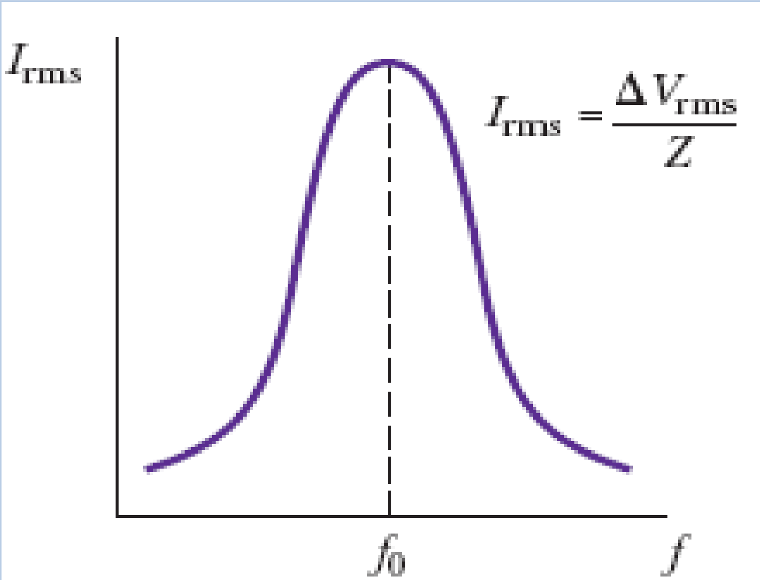
و عندها يصبح

$$Z = R$$

يحدث ذلك عند تردد f_0 يسمى **تردد الرنين**

Resonance in a Series RLC Circuit الرنين في دوائر RLC توالي

التيار دالة في التردد لدئرة تحوي قيمة ثابتة لكل من المكثف و الملف



$$X_L = X_C$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The tuning circuit of a radio is an important application of a series resonance circuit. The radio is tuned to a particular station (which transmits a specific radiofrequency signal) by varying a capacitor, which changes the resonance frequency of the tuning circuit. When this resonance frequency matches that of the incoming radio wave, the current in the tuning circuit increases.

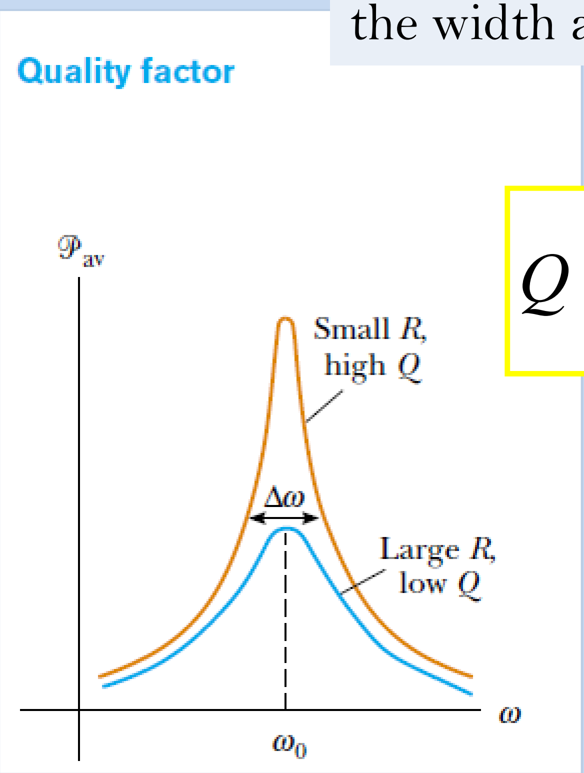
Resonance in a Series RLC Circuit الرنين في دوائر RLC توالي

This curve sharpness is usually described by a dimensionless parameter known as the quality factor, denoted by *Q*:

$$Q = \frac{\omega_0}{\Delta\omega}$$

the width at the half-power points has the value

$$\Delta\omega = R/L$$



$$Q = \frac{\omega_0 L}{R}$$

Worksheet:2

1- What is the impedance of a series *RLC circuit at resonance?*

(a) larger than *R*

(b) less than R

(c) equal to R

(d) impossible to determine

Example 33.3 A Purely Capacitive AC Circuit

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution Using Equation 33.18 and the fact that $\omega = 2\pi f = 377\ \text{s}^{-1}$ gives

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = 332\ \Omega$$

Hence, from a modified Equation 33.19, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\ \text{V}}{332\ \Omega} = 0.452\ \text{A}$$

What If? **What if the frequency is doubled? What happens to the rms current in the circuit?**

Answer If the frequency increases, the capacitive reactance decreases—just the opposite as in the case of an inductor. The decrease in capacitive reactance results in an increase in the current.

Let us calculate the new capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = 166\ \Omega$$

The new current is

$$I_{\text{rms}} = \frac{150\ \text{V}}{166\ \Omega} = 0.904\ \text{A}$$

Example 33.5 Analyzing a Series *RLC* Circuit

A series *RLC* AC circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ s^{-1}$, and $\Delta V_{max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution The reactances are $X_L = \omega L = 471\ \Omega$ and $X_C = 1/\omega C = 758\ \Omega$.

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = 513\ \Omega$$

(B) Find the maximum current in the circuit.

Solution

$$I_{max} = \frac{\Delta V_{max}}{Z} = \frac{150\text{ V}}{513\ \Omega} = 0.292\text{ A}$$

(C) Find the phase angle between the current and voltage.

Solution

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right)$$
$$= -34.0^\circ$$

Because the capacitive reactance is larger than the inductive reactance, the circuit is more capacitive than inductive. In this case, the phase angle ϕ is negative and the current leads the applied voltage.