1) $\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\, dx$

Soln:- where $\cos 2x = 1 - 2\sin^2 x$

$\therefore \cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\therefore I = \int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\, dx = \int_0^{2\pi} \sqrt{\frac{1-\left(1-2\sin^2\left(\frac{x}{2}\right)\right)}{2}}\, dx$

$= \int_0^{2\pi} \sqrt{\frac{\cancel{1}-\cancel{1}+\cancel{2}\sin^2\left(\frac{x}{2}\right)}{\cancel{2}}}\, dx$

$= \int_0^{2\pi} \sqrt{\sin^2\left(\frac{x}{2}\right)}\, dx = \int_0^{2\pi} \sin\left(\frac{x}{2}\right) dx$

$= 2\int_0^{2\pi} \left(\frac{1}{2}\right) \sin\left(\frac{x}{2}\right) dx$

$= 2\left(-\cos\left(\frac{x}{2}\right)\right)_0^{2\pi} = 2\left[-(-1) + (1)\right]$

$= 4.$ #

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$\boxed{2}$ $\displaystyle\int_0^{\pi/6} \sqrt{1+\sin x}\, dx$

$\underline{\text{Soln}}$:- By multiplying by $\sqrt{\dfrac{1-\sin x}{1-\sin x}}$

$$\therefore \underline{I} = \int_0^{\pi/6} \sqrt{1+\sin x} \cdot \sqrt{\frac{1-\sin x}{1-\sin x}}\, dx$$

$$= \int_0^{\pi/6} \sqrt{\frac{1-\sin^2 x}{1-\sin x}}\, dx, \quad \text{where } \cos^2 x + \sin^2 x = 1$$

$$= \int_0^{\pi/6} \sqrt{\frac{\cos^2 x}{1-\sin x}}\, dx = \int_0^{\pi/6} \frac{\cos x}{\sqrt{1-\sin x}}\, dx$$

$$= -\int_0^{\pi/6} (1-\sin x)^{-1/2} \cdot (-\cos x)\, dx$$

$$= \Big[-2\sqrt{1-\sin x}\Big]_0^{\pi/6}$$

$$= -2\sqrt{1-\sin(30^\circ)} + 2\sqrt{1-\sin 0^\circ} = \frac{-2}{\sqrt{2}} + 2$$

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$\boxed{3}$ $\displaystyle\int_{\pi/4}^{\pi/2} \csc^4\theta \, d\theta$

<u>Soln:-</u> where $\csc^2\theta = 1 + \cot^2\theta$

$$\therefore I = \int_{\pi/4}^{\pi/2} \csc^4\theta \, d\theta = \int_{\pi/4}^{\pi/2} \csc^2\theta \cdot \csc^2\theta \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} (1+\cot^2\theta) \csc^2\theta \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \csc^2\theta \, d\theta + \int_{\pi/4}^{\pi/2} \cot^2\theta \csc^2\theta \, d\theta$$

$$= \Big[-\cot\theta\Big]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} (\cot\theta)^2 (-\csc^2\theta) \, d\theta$$

$$= \Big[-0+1\Big] - \frac{1}{3}\Big[(\cot\theta)^3\Big]_{\pi/4}^{\pi/2}$$

$$= 1 + \frac{1}{3} = \frac{4}{3} \quad \#$$

(4) $\int_{\pi/6}^{\pi/3} \cot^3 x \, dx$

Sol: Where $1+\cot^2 x = \csc^2 x$.

$\cot^2 x = \csc^2 x - 1$

$$\therefore I = \int_{\pi/6}^{\pi/3} \cot^3 x \, dx = \int_{\pi/6}^{\pi/3} \cot x \cdot \cot^2 x \, dx$$

$$= \int_{\pi/6}^{\pi/3} (\csc^2 x - 1) \cot x \, dx$$

$$= -\int_{\pi/6}^{\pi/3} \cot x \, (-\csc^2 x) dx - \int_{\pi/6}^{\pi/3} \cot x \, dx$$

$$= -(\cot^2 x)\Big|_{\pi/6}^{\pi/3} \cdot \frac{1}{2} - \Big[\ln|\sin x|\Big]_{\pi/6}^{\pi/3}$$

$$= -\frac{1}{2}\left(\frac{1}{3} - 3\right) - \left[\ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{8}^{4}}{3} - \ln(\sqrt{3}) \quad \#$$

5 $\int e^{\sqrt{3s+a}}\,ds$

Soln.: Let $z^2 = 3s+a$

By Diff., $2z\,dz = 3\,ds$

$\therefore ds = \frac{2}{3} z\,dz$

$$\therefore I = \int e^{\sqrt{3s+a}}\,ds = \int e^{\sqrt{z^2}} \cdot \frac{2}{3} z\,dz$$

$$= \frac{2}{3} \int z e^{z}\,dz$$

$\Downarrow$ integral by parts.

let $u = z$ $\qquad dv = e^{z}\,dz$

$du = dz$ $\qquad v = e^{z}$

$$\therefore I = \frac{2}{3}\left[z e^{z} - \int e^{z}\,dz \right]$$

$$I = \frac{2}{3}\left[z e^{z} - e^{z}\right]$$

where $z = \sqrt{3s+a}$

$$\therefore I = \frac{2}{3}\left[\sqrt{3s+a}\, e^{\sqrt{3s+a}} - e^{\sqrt{3s+a}}\right]$$

$$= \frac{2}{3} e^{\sqrt{3s+a}}\left[\sqrt{3s+a} - 1\right]$$

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