Exercise :- Use the Mathematical Induction to Prove that :- $\forall n \geq 5 \quad n < 2^n$.

Proof:

1 (Basic of induction)

We want to show that this statement is true at $(n=5)$.

$\therefore$ at $n=5 \Rightarrow 2^5 = 32 \Rightarrow 5 < 32$

$\therefore$ the relation is correct at $n=5$.

2 (Induction hypothesis)

Assume that the relation is correct at n

i.e. $\forall n \geq 5 \Rightarrow n < 2^n$.

3 (Induction step)

we want to show that this relation is true at $(n+1)$. i.e. $n+1 < 2^{(n+1)}$?

when $2^{(n+1)} = 2^n \cdot 2$. But From 2 $\Rightarrow 2^n > n$

Then $2^{(n+1)} > 2n$, But $2n = n+n$

and $n \geq 5 \Rightarrow n+n \geq n+1$

So, we have

$$2^{(n+1)} \geq n+1 \quad \#$$

i.e. the relation is true at $(n+1)$.

From [1], [2], [3] we have that this relation is true For all $n \geq 5$.